Computer Science 3711 / Engineering 5891
Winter 2001

Class Exam

February 20, 2001

Instructor:
T. Wareham

NAME: _____ STUDENT ID #: _____

- This exam will be 65 minutes long.
- Please answer all questions given in this exam.
- Please answer all questions in the space provided on this exam; if you find it necessary to continue an answer on the back of a sheet of paper, that is fine, but please make a note on the front side, *e.g.*, "answer cont'd on back".

Question	Mark
1.	
2. a)	
b)	
3. a)	
b)	
4. a)	
b)	
5. a)	
b)	
c)	
TOTAL	

1. (8 marks)

Give an **exact** closed form, *i.e.*, without summations, expression for

$$(\sum_{k=0}^{n} k(k+3)) - (\sum_{k=3}^{n} k^2)$$

2. (12 marks)

a) (6 marks) Show that $T(n) \leq n - 1$ by the substitution method, where

$$T(n) = \begin{cases} 0 & n < 3 \\ 3T(\lfloor \frac{n}{3} \rfloor) + 2 & n \geq 3 \end{cases}$$

b) (6 marks) Derive an upper bound for $T(n)$ by the iteration method, where

$$T(n) = \begin{cases} 0 & n \leq 1 \\ 3T(n-1) + 2 & n > 1 \end{cases}$$

3. (10 marks)

As part of the proofs below, please give appropriate c- and n_0-values.

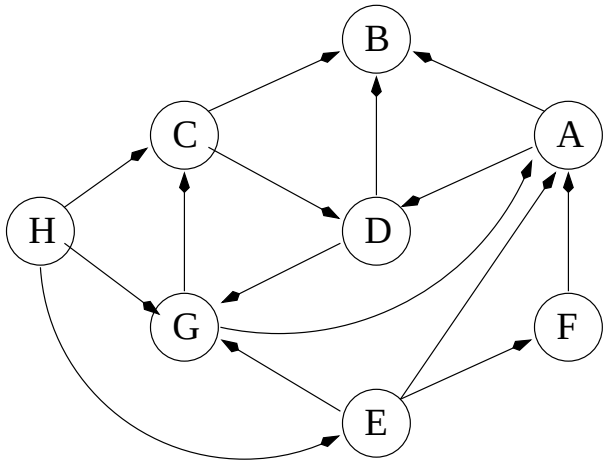
a) (6 marks) Prove that $f(n) = (n-5)^2$ is $\Theta(n^2)$.

b) (4 marks) Prove that for any graph $G = (V, E)$, $f(G) = \frac{|E|}{|V|} \log |E|$ is $O(|V| \log |V|)$.

4. (15 marks)

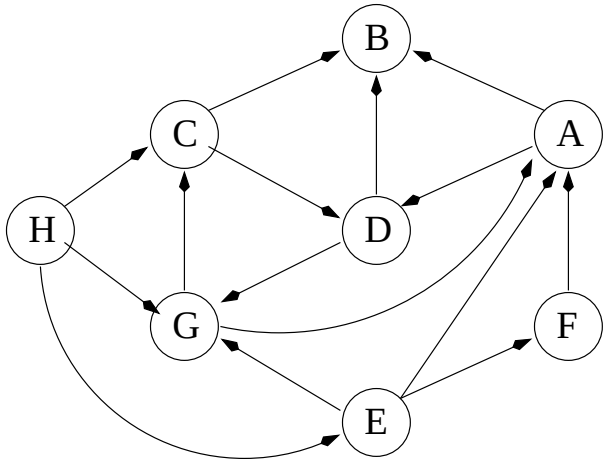
Assume that the algorithms cited below consider vertices in alphabetical order and that each adjacency list is ordered alphabetically.

a) (5 marks) Consider the following directed graph:



Give the graph at the end of the execution of the BFS algorithm when the search is started at vertex D, with the d-values for all vertices as well as all BFS-search tree edges clearly marked.

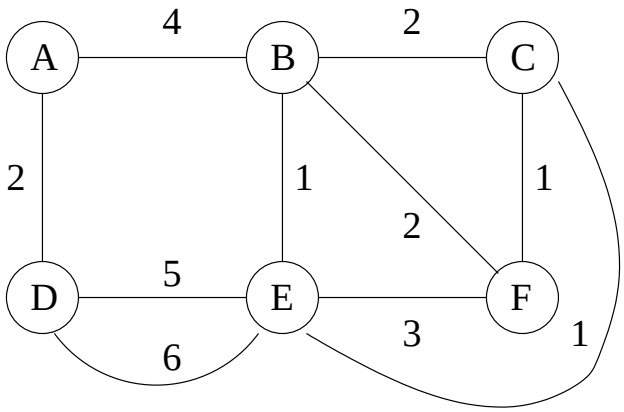
b) (10 marks) Consider the following directed graph:



Give the graph at the end of the execution of the DFS algorithm, with the d- and f-values of all vertices as well as the types of all edges clearly marked.

5. (20 marks)

a) (5 marks) Consider the following weighted undirected graph:



Give the graph at the end of the execution of Kruskal's minimum spanning tree algorithm, with all tree-edges and the order in which each tree-edge was added clearly marked.

b) (6 marks) Suppose the graph in part (a) is modified to contain an additional edge (B, D) with weight (i) 6, (ii) 4, or (iii) 2. For each of these three cases, say whether or not Kruskal's algorithm could produce a different minimum spanning tree from that given in part (a) of this question, as well as whether or not the number of minimum spanning trees of the modified graph is different from the number of minimum spanning trees of the original graph in part (a).

i) Weight = 6 Different min spanning tree possible under Kruskal? _____

\# min spanning trees different? _____

ii) Weight = 4 Different min spanning tree possible under Kruskal? _____

\# min spanning trees different? _____

iii) Weight = 2 Different min spanning tree possible under Kruskal? _____

\# min spanning trees different? _____

c) (9 marks) Consider the following simplified form of Prim's minimum spanning tree algorithm:

```
INIT_Q(V);
pred[r] = nil;
while NOT_EMPTY(Q) do
    u = EXTRACT_MIN(Q);
    for each v that is adjacent to u in G do
        if IN_Q(v) and UPDATE_Q(v) then
            pred[v] = u;
```

As discussed in class and described in the textbook, this algorithm runs in $O(|E| \log |V|)$ time if priority queue Q is implemented as a binary heap, which allows the operations INIT_Q, NOT_EMPTY, EXTRACT_MIN, IN_Q, and UPDATE_Q be done in $O(|V|)$, $O(1)$, $O(\log |V|)$, $O(1)$, and $O(\log |V|)$ time, respectively. Give the time complexity of Prim algorithm when the INIT_Q and and NOT_EMPTY operations require $O(|V|)$ and $O(1)$ time, respectively, and the EXTRACT_MIN, IN_Q, and UPDATE_Q operations require

i) $O(|V|)$, $O(|V|)$, and $O(1)$ time, respectively;

ii) $O(|V|)$, $O(1)$, and $O(|V|)$ time, respectively; and

iii) $O(1)$, $O(\log |V|)$, and $O(|V|)$ time, respectively