Computer Science 3711 (Fall 2005):
Bonus Assignment
Due: 5:00 PM on Tuesday, December 13, 2005

1. **(10 marks)** For each of the situations described in the parts of this question, state the corresponding problem in terms of its inputs and outputs along the lines of the statement of the sorting problem on page 5 of the textbook.

   (5 marks) A hospital oversight committee is evaluating various proposed designs for an isolation ward with respect to susceptibility to airborne disease outbreaks. Each isolation ward design consists of a set of rooms linked in a pairwise fashion by airducts. Each airduct has a particular configuration of viral filters and hence a particular probability of transmitting a viral disease from one room to the other. Given a set A of airducts connecting a set R of rooms, the probability of a disease spreading to all of the rooms in R is the sum of the transmission-probabilities of the airducts in A. Given such a design and a specified room in which a patient with an airborne disease has been placed, you want to know if that disease can spread to all rooms in the ward with probability 1.

   b) (5 marks) Earlier today, your favorite nephew dropped by your mansion and asked to borrow your Rolls-Royce so he could get groceries. You lent him your car and asked him to pick up a bag of piranha food for the guest house as well while he was out. Your nephew returned three and a half-hours later with the groceries and piranha food, claiming that he was delayed because he had to go to multiple stores to pick up everything and that he didn't go anywhere else. You know the distances between all stores in your city that carry groceries and/or piranha food as well as the distances between those stores and your mansion; moreover, as you don't quite trust your nephew, you wrote down the mileage on your car immediately before and after your nephew borrowed it. You want to know if it is possible that your nephew is telling the truth.

2. **(90 marks)** For each of the following problems, either prove that this problem is in P (by giving a polynomial-time algorithm) or prove that this problem is NP-complete (by showing that this problem is in NP and giving a polynomial-time many-one reduction (algorithm + two-way proof of correctness) to this problem from an NP-hard problem).

   Hints:

   * Three of the problems below are solvable in polynomial-time and three are NP-complete.
   * When showing NP-hardness for the problems below, you may find it convenient to use reductions from some of the following problems: 0/1 Knapsack, 3-CNF (p. 999), 3-Colorability (p. 1019), Clique (p. 1003), Hamiltonian cycle (p. 1008), Longest path (p. 978), and Vertex cover (p. 1006).

(a) **(15 marks)**

c-ZPath (cZP) (c is an integer constant of value ≥ 1)
Input: An undirected graph $G = (V, E)$.
Question: Can the vertices in G be colored with two colors such that (1) no edge's endpoint-vertices have the same color and (2) there is a path in this colored version of G with $\geq c$ edges in which no vertex or edge repeats and the vertex-colors alternate for the entire length of the path?

(b) **(15 marks)**

Bi-Robustness (BR)
Input: An undirected graph $G = (V, E)$, a pair of vertices $x, y \in V$, and a positive integer k.
Question: Are there two paths P_1 and P_2 linking x and y in G such that each path contains at least k edges and $P1$ and $P2$ do not intersect, *i.e.*, $P1$ and $P2$ do not have any edges in common, no vertex or edge occurs more than once in $P1$ or $P2$, and the only vertices $P1$ and $P2$ have in common are x and y?

(c) **(15 marks)**

Hypergraph 4-Colorability (H4C)
Input: A set of items U and a set $S = \{S_1, S_2, \ldots, S_n\}$ of subsets of U such that $|S_i| = 3$ for $1 \leq i \leq n$.
Question: Given the set of colours {*red, blue, green, yellow*}, can we color the items in U such that there is no subset S_i, $1 \leq i \leq n$, in which two of the three items in S_i have the same color?

(d) **(15 marks)**

c-Accumulability (cA) (c is an integer constant of value ≥ 1)
Input: A directed graph $G = (V, A)$ a set of subsets $S = \{S_1, S_2, \ldots, S_c\}$ of V, a pair of vertices $x, y \in V - (\bigcup_{1=1}^{c} S_i)$, and a positive integer k.
Question: Is there a path in G from x to y such that this path includes at least one vertex from each of the subsets in S and the number of edges in this path is $\leq k$?

(e) **(15 marks)**

k-PClique (kPC)
Input: An undirected graph $G = (V, E)$ and a positive integer k.
Question: Is there a subset $V' \subseteq V$ such that $|V'| \geq k$ and there is a path in G between each pair of vertices in V'?

(f) **(15 marks)**

0/1 Nested Knapsack (01NK)
Input: A set of items U, item size- and value-function $s()$ and $v()$, a positive integer k, and positive integers B_1 and B_2 such that $B_1 \leq B_2$.
Question: Are there subsets U_1 and U_2 of U such that $U_1 \subset U_2$, $\sum_{u \in U_1} s(u) \leq B_1$, $\sum_{u \in U_2} s(u) \leq B_2$, and $\sum_{u \in U_2} v(u) \geq k$?