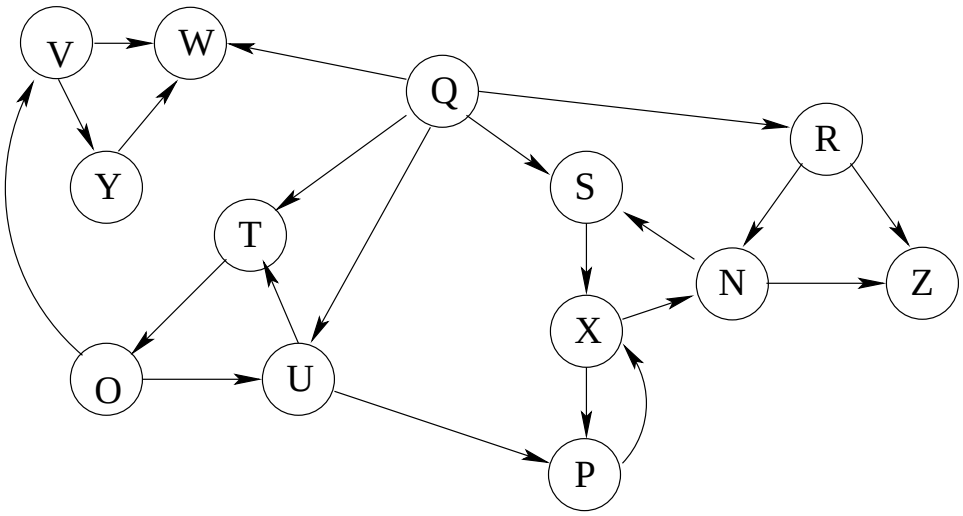
Computer Science 3711 (Fall 2005):
Assignment #4, Questions #2–4
Due: 4:00 PM on Wednesday, November 16, 2005

2. (15 marks) Consider the following directed graph:



Assume that the algorithms below consider vertices in alphabetical order and that each adjacency list is ordered alphabetically.

a) (5 marks) Show how breadth-first search works on this graph. Give the graph at the end of the execution of the BFS algorithm on page 532 of the textbook when the search is started at vertex X, with the d- and π-values for all vertices as well as all BFS-search tree edges clearly marked.

b) (5 marks) Redo part (a) above with the BFS algorithm starting at vertex Q.

c) (5 marks) Show how depth-first search works on this graph. Give the graph at the end of the execution of the DFS algorithm on page 541 of the textbook, with the d-, f-, and π-values of all vertices as well as the types of all edges clearly marked.

3. (25 marks) For each of the problems below, give a pseudocode algorithm and a parameterized asymptotic worst-case time complexity for that algorithm. Note that your algorithm must run such that when the running-time parameter for the given procedure is polynomial, the parameterized time complexity expression is also polynomial.

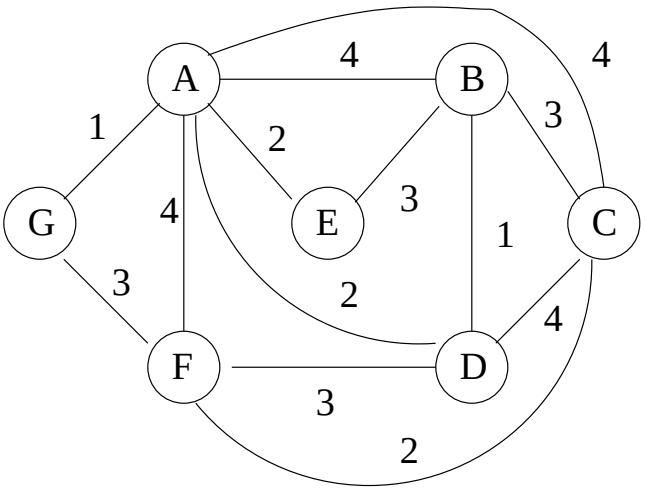
a) (10 marks) Given an undirected graph $G = (V, E)$ and an integer $k > 0$, determine if G is k-frangible, *i.e.*, is there a subset $V' \subseteq V$, $|V'| \leq k$, such that for each vertex $v \in V$, either $v \in V'$ or v is adjacent (via an edge in E) to a vertex in V'? You may use the following operations:

- `SCDec(S, I, k)`: Returns `true` if there is set-cover of size $\leq k$ for I in S and `false` otherwise.
- Basic operations on graphs, *e.g.*, graph-creation, operations on vertex- and edge-sets (see below).
- Basic operations on sets, *e.g.*, set-creation, set-size, item-membership, item-addition, item-deletion, set-union, set-intersection, set-item-enumeration.

b) (15 marks) Given an item-set I and a set S of subsets of I, compute a minimum-size set cover for I. You may use the following operations:

- `SCDec(S, I, k)`: Returns `true` if there is set-cover of size $\leq k$ for I in S and `false` otherwise.
- Basic operations on sets, *e.g.*, set-creation, set-size, item-membership, item-addition, item-deletion, set-union, set-intersection, set-item-enumeration.

4. (10 marks) Consider the following weighted undirected graph:



a) (5 marks) Show how Kruskal's minimum spanning tree algorithm works on this graph. Give the graph at the end of the execution of the algorithm on page 569 of the textbook, with all tree-edges and the order in which each tree-edge was added clearly marked.

b) (5 marks) Show how Prim's minimum spanning tree algorithm works on this graph relative to root-vertex $r = \mathrm{G}$. Give the graph at the end of the execution of the algorithm on page 572 of the textbook, with all tree-edges and the order in which each tree-edge was added clearly marked.