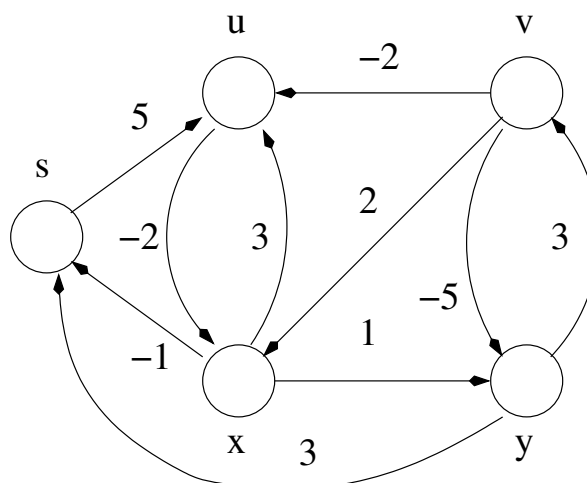


Computer Science 3600 (Winter 2023):
 Assignment #3, Questions #2–4
 Due: 11:59 PM on Thursday, April 6, 2023

1. (14 marks) Determine the optimal parenthesization of a matrix-chain product whose sequence of dimensions is $\langle 2, 5, 3, 4, 2, 3, 5 \rangle$ using the algorithms given on page 375 and 377 of the textbook. In the style of Figure 15.5, show the filled-in dynamic programming matrices m and s , the “backpointer path” in s that gives an optimal parenthesization, and the parenthesization associated with that path.
2. (20 marks) Consider the following edge-weighted directed graph:



- a) (10 marks) Run Dijkstra’s algorithm (p. 658) on the directed graph above using vertex x as the source vertex. In the style of Figure 24.6 in the textbook, show the d and π values and the vertices in set S after each iteration of the **while** loop.
- b) (10 marks) Run the Bellman-Ford algorithm (p. 651) on the directed graph above using vertex x as the source vertex. Relax edges in lexicographic order in each pass, and in the style of Figure 24.4 on the textbook, show the d and π values after each pass. Finally, give the boolean value returned by the algorithm.
3. (16 marks) Consider the following decision problems:

DUMBBELL SUBGRAPH (DS)

Input: An undirected graph $G = (V, E)$ and two positive integers $k, l \geq 1$.

Question: Are there two cliques C_1 and C_2 and a simple path P in G such that C_1 and C_2 have $\geq k$ vertices apiece, P has $\geq l$ edges, P connects C_1 and C_2 , the cliques and path do not have any edges in common, and the only vertices that P shares with C_1 (C_2) is its connection-vertex?

BOUNDED-WEIGHT SUBSET COVER (BWSSC)

Input: A set $I = \{i_1, \dots, i_a\}$ of items, a set $R = \{r_1, \dots, r_b\}$ of subsets of I , an integer-valued subset-weight function $w()$ such that for each $r_x \in R$, $w(r_x) > 0$, a subset $N \subseteq I$, and integers $0 < k_1 \leq k_2$.

Question: Is there a subset $R' \subseteq R$ such that $\cup_{r \in R'} r = N$ and $k_1 \leq \sum_{r \in R'} w(r) \leq k_2$?

- a) (8 marks) Prove that problem DS is *NP*-complete by (1) showing that this problem is in *NP* and (2) giving a polynomial-time many-one reduction (algorithm + proof of correctness) to this problem from the following *NP*-hard problem:

HAMILTONIAN PATH (HP)

Input: An undirected graph G .

Question: Is there a simple path in G that links all vertices in G ?

- b) (8 marks) Prove that problem BWSSC is *NP*-complete by (1) showing that this problem is in *NP* and (2) giving a polynomial-time many-one reduction (algorithm + proof of correctness) to this problem from the following *NP*-hard problem:

SET COVER (SC)

Input: A set $U = \{u_1, \dots, u_n\}$ of items, a set $S = \{s_1, \dots, s_m\}$ of subsets of U , and an integer $k \leq m$.

Question: Is there a subset $S' \subseteq S$ such that $|S'| \leq k$ and $\cup_{s \in S'} s = U$?