Digital Circuits

DIGITAL CIRCUITS: AN OVERVIEW

BOOLEAN LOGIC

DIGITAL CIRCUIT DESIGN

IMPLEMENTING DIGITAL CIRCUITS
Digital Circuits: An Overview

- Given binary memory, need to build digital circuits that implement computational operations (cf. analog circuits).
- Digital circuit as $n$-input $\rightarrow m$-output transformation:

- Focus here on combinatorial circuits that do not involve feedback (cf. feedback-based sequential circuits).
Two types of circuits: \textbf{arithmetic} and \textbf{control}.
Specify circuit in terms of input-output behavior, e.g.,

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$B$</td>
</tr>
<tr>
<td>0</td>
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</table>

... But how do we design circuits from specifications? ...
Boolean Logic: An Overview

- Self-taught mathematician.
- 1854 book *The Laws of Thought* developed algebraic approach to logic (*Boolean logic*); part of algebraic formalization of other areas of mathematics, e.g., geometry, probability.
- Based on variables with values *True* or *False* and three operators: AND (\( \cdot \)), OR (\( + \)), and NOT (\( \overline{A} \)).

George Boole
(1815-1864)
Specify behavior of operators as **truth tables**.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>A AND B</th>
<th>A OR B</th>
<th>NOT A</th>
</tr>
</thead>
<tbody>
<tr>
<td>False</td>
<td>False</td>
<td>False</td>
<td>False</td>
<td>True</td>
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<td>True</td>
<td>True</td>
<td>True</td>
<td>True</td>
<td>False</td>
</tr>
</tbody>
</table>
Boolean Logic: An Overview (Cont’d)

• Operators and variables can be combined to create expressions, e.g.,

\[(\overline{(A \cdot B)} + \overline{C})\]

\[\text{NOT } ((A \text{ AND } B) \text{ OR } (\text{NOT } C))\]

• Each expression in turn has an associated truth table, which is created by applying the operators in the expression to each possible combination of values for the variables in the expression, e.g.,

\[A = \text{False}, B = \text{True}, C = \text{False} \implies \text{False}\]

• What about deriving an expression for a given truth table?
OR together the AND-expressions corresponding to variable values in the rows of the truth table that yield result *True*, e.g.,

\[
\begin{array}{cccc}
A & B & C & \text{RESULT} \\
\text{False} & \text{False} & \text{False} & \text{False} \\
\text{False} & \text{False} & \text{True} & \text{True} \\
\text{False} & \text{True} & \text{False} & \text{False} \\
\text{False} & \text{True} & \text{True} & \text{True} \\
\text{True} & \text{False} & \text{False} & \text{False} \\
\text{True} & \text{False} & \text{True} & \text{True} \\
\text{True} & \text{True} & \text{False} & \text{False} \\
\text{True} & \text{True} & \text{True} & \text{False} \\
\end{array}
\]

\[
\implies (\overline{A} \cdot \overline{B} \cdot C) + (\overline{A} \cdot B \cdot C) + (A \cdot \overline{B} \cdot C)
\]
Digital Logic Design: Beginnings

Claude Shannon (1916-2001)

Electromechanical Telephone Switch (1930s)
• Shannon MSc thesis (MIT, 1937): Boolean logic can be used to design telephone switching networks!
• Let 0 and 1 in circuits correspond to $False$ and $True$ and abstract logic gates to Boolean operators.
Digital Logic Design: Beginnings (Cont’d)

Represent network circuits as logic gate diagrams, e.g.,

Boolean expression $\iff$ logic gate diagram
Truth table $\iff$ logic gate behavior specification
Digital Circuit Design: The Sum-of-Products Algorithm

Get behaviour specification for circuit
for each output column in specification do
  Construct AND subexpressions for rows
  with output 1
  Use ORs to combine the constructed AND
  subexpressions
  Create logic circuit diagram corresponding to
  OR expressions
Digital Circuit Design:  
The Sum-of-Products Algorithm (Cont’d)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>C\text{IN}</th>
<th>SUM</th>
<th>C\text{OUT}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
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</tbody>
</table>

\[
\text{SUM} = (\overline{A} \cdot \overline{B} \cdot C_{\text{IN}}) + (A \cdot \overline{B} \cdot \overline{C}_{\text{IN}}) + (A \cdot \overline{B} \cdot C_{\text{IN}}) + (A \cdot B \cdot C_{\text{IN}})
\]

\[
\text{C}_{\text{OUT}} = (\overline{A} \cdot B \cdot C_{\text{IN}}) + (A \cdot \overline{B} \cdot C_{\text{IN}}) + (A \cdot B \cdot \overline{C}_{\text{IN}}) + (A \cdot B \cdot C_{\text{IN}})
\]
Arithmetic Circuit Design: 1-Bit Adder

Figure 4.26 Complete 1-ADD Circuit for 1-Bit Binary Addition
Can dramatically decrease the number of gates by applying rules of Boolean algebra and using advanced logic gates such as XOR (Exclusive OR), e.g.,
Arithmetic Circuit Design: \( n \)-Bit Adder

**Figure 4.27** The Complete Full Adder ADD Circuit
Arithmetic Circuit Design: 1-Bit Compare-For-Equality

- This type of circuit returns 1 if the two given bits $A$ and $B$ have the same value.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>RESULT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
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</table>

$$RESULT = (\overline{A} \cdot \overline{B}) + (A \cdot B)$$

- Again, there are several implementations of this circuit.
Arithmetic Circuit Design: 1-Bit Compare-For-Equality (Cont’d)

Figure 4.22
One-Bit Compare for Equality Circuit
Arithmetic Circuit Design: \( n \)-Bit Compare-For-Equality

N-bit Compare for Equality Circuit
Control Circuits: Overview

- Control circuits determine the order in which operations are carried and select the correct data values to be processed → they are sequencing and decision-making circuits.
- Two main types:
  1. **Multiplexor**: Use $N$ input selector lines to determine which of $2^N$ input value lines has its value copied to the single output line (see Textbook, Figure 4.33, p. 211).
  2. **Decoder**: Use $N$ input selector lines to determine which of $2^N$ output lines is set to 1 (with all others being set to 0) (see Textbook, Figure 4.35, p. 213).
Control Circuit Design:
Multiplexor (Abstract)

2^N Input Value Lines

Multiplexor

1 Output Line

N Input Selector Lines
Control Circuit Design:
Two-input Multiplexor Circuit

Figure 4.28
A Two-Input Multiplexor Circuit
Control Circuit Design: Data-routing Multiplexor

**Figure 4.31** Example of the Use of a Multiplexor Circuit
Control Circuit Design: Decoder (Abstract)

N Input Lines 2^N Output Lines

Decoder
Control Circuit Design:
2-to-4 Decoder Circuit

Figure 4.29
A 2-to-4 Decoder Circuit
Control Circuit Design: Op-code Decoder

Figure 4.30 Example of the Use of a Decoder Circuit
Implementing Digital Circuits: Transistors

Transistor (1947)  Transistor Board
Implementing Digital Circuits: Silicon Chips

Integrated Circuit (IC) (1959)  IC Internals ("Chip")

Moore’s Law (1965): 2x Transistor Density Every 18 Months
And If You Liked This...

- MUN Computer Science courses on this area:
  - COMP 1002: Introduction to Logic for Computer Scientists
  - COMP 2003: Computer Architecture
  - COMP 4723: Introduction to Microprocessors
- MUN Computer Science professors teaching courses / doing research in this area:
  - Miklos Bartha
  - Rod Byrne
  - Ashoke Deb
  - Antonina Kolokolova
  - Manrique Mata-Montero