Systematic Parameterized Complexity Analysis in Computational Phonology

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Abstract

Many computational problems are NP-hard and hence probably do not have fast, i.e., polynomial time, algorithms. Such problems may yet have non-polynomial time algorithms, and the non-polynomial time complexities of these algorithms will be functions of particular aspects of that problem, i.e., the algorithm's running time is upper bounded by $f(k)|x|^c$, where f is an arbitrary function, $|x|$ is the size of the input x to the algorithm, k is an aspect of the problem, and c is a constant independent of $|x|$ and k. Given such algorithms, it may still be possible to obtain optimal solutions for large instances of NP-hard problems for which the appropriate aspects are of small size or value. Questions about the existence of such algorithms are most naturally addressed within the theory of parameterized computational complexity developed by Downey and Fellows.

This thesis considers the merits of a systematic parameterized complexity analysis in which results are derived relative to all subsets of a specified set of aspects of a given NP-hard problem. This set of results defines an "intractability map" that shows relative to which sets of aspects algorithms whose non-polynomial time complexities are purely functions of those aspects do and do not exist for that problem. Such maps are useful not only for delimiting the set of possible algorithms for an NP-hard problem but also for highlighting those aspects that are responsible for this NP-hardness.

These points will be illustrated by systematic parameterized complexity analyses of problems associated with five theories of phonological processing in natural languages – namely, Simplified Segmental Grammars, finite-state transducer based rule systems, the KIMMO system, Declarative Phonology, and Optimality Theory. The aspects studied in these analyses broadly characterize the representations and mechanisms used by these theories. These analyses suggest that the computational complexity of phonological processing depends not on such details as whether a theory uses rules or constraints or has one, two, or many levels of representation but rather on the structure of the representation-relations encoded in individual mechanisms and the internal structure of the representations.