Texture Mapping

Outline
- What is texture mapping?
- What texture can be used for?
- How to do texture mapping?
  - Parameterization + Sampling
  - Bump mapping

What is Texture Mapping?
- Texture mapping is the process of mapping a point on a geometric model to a location on texture map
- A technique used to paste an image onto a geometric model to provide visual details

How to Represent a Complex Object?
- Expensive solution:
  - Use a complex polygon mesh to model both the shape & the shading details of the object
  - Different facets have different shading details
- Efficient solution:
  - Use simple polygon mesh to model the shape of the object only
  - Use images to model the changes of shading details
  - Map the images onto the polygon mesh

Details Handled By Textures
- Color changes:
  - Diffuse color; Specular color
- Reflectance property changes:
  - Specular exponent; Transparency
- Geometric changes:
  - Normal; Displacement
- Lighting changes:
  - Lighting pattern; Shadow

2D Textures
- Function:
  - \( C = T(u, v) \)
- Image texture:
  - Scan a painting
  - Take a picture of the wood, marble, sky, etc.
- Procedural texture:
  - Generated using a program
Procedural Texture Example

• Checkerboard pattern:
  • int p = u / size;
  • int q = v / size;
  • if ( (p+q) % 2 == 0 )
    • return Color.Red;
  • else
    • return Color.White;

How to Do Texture Mapping

• Step 1: Parameterization
  • Map a 3D point (x,y,z) on the geometry to 2D parameter (u,v) on an abstract unit square

• Step 2: Sampling
  • Map parameter (u,v) on abstract unit square to a pixel (i,j) on the image

Sampling Step

• Transform a point on the unit square to a position in the texture image
  • A 2D transformation
  • Sample the color of the texture image at the transformed position
  • Avoid over-sampling or under-sampling problems

Parameterization Step

• How to map a 3D location (x,y,z) to a 2D parameters (u,v)
• Some objects have natural ways for parameterizations:
  • Such as sphere or cylinder
  • Arbitrary polygon meshes are more difficult

Different Sampling Effects

• (u,v) ->
  • u * width,
  • (1-v) * height
• (u,v) ->
  • (u*width*2) % width,
  • ((1-v)*height*3) % height

Mapping of a Sphere

• Use spherical coordinates:
  • u: latitude
  • v: longitude
• Calculation:
  • θ = atan2(z,x)
  • ϕ = asin(y/r)
  • u = θ / 2π + 0.5
  • v = ϕ / π + 0.5
• Problem:
  • Singular points at two poles
Mapping of a Cylinder
- Use cylindrical coordinates:
- Calculation:
  - $\theta = \text{atan2}(z,x)$
  - $u = \theta / 2\pi + 0.5$
  - $v = y / \text{height}$

Mapping of a Polygon Mesh I
- Use an intermediate object:
  - Such as sphere
- Two steps:
  - Project point on polygon mesh to a point on the intermediate object
  - Calculate texture parameters using the intermediate object

Mapping of a Polygon Mesh II
- Design the mapping relation manually
- Define the texture coordinate for each vertex in the polygon mesh
- The texture coordinate for an arbitrary point on the mesh can then be interpolated

Texture Mapping Example

Bump Mapping
- Traditional texture mapping changes the color of surface only
  - The object appears flat
- Bumping mapping:
  - Proposed by Blinn in 1978
  - Use texture image to perturb surface normals
  - Silhouette edges still appear smooth

2D Bump Map
- 2D height field array:
  - Keep the distance between the model & the desired bumpy surface along the normal
  - $F' = F + B \times N$
  - $B$: the bump map
  - $F$: the surface defined by the model
  - $F'$: the desired bumpy surface
Normal Perturbation

\[ F'_{e,j} = F_{e,j} + B_{e,j} N_{e,j} \]

\[ \frac{\partial F}{\partial u} \bigg|_{e,j} = \frac{\partial F}{\partial u} \bigg|_{e,j} + \frac{\partial B}{\partial u} \bigg|_{e,j} N_{e,j} + B_{e,j} \frac{\partial N}{\partial u} \bigg|_{e,j} = \frac{\partial B}{\partial u} \bigg|_{e,j} N_{e,j} \]

\[ \frac{\partial F}{\partial v} \bigg|_{e,j} = \frac{\partial F}{\partial v} \bigg|_{e,j} + \frac{\partial B}{\partial v} \bigg|_{e,j} N_{e,j} + B_{e,j} \frac{\partial N}{\partial v} \bigg|_{e,j} = \frac{\partial B}{\partial v} \bigg|_{e,j} N_{e,j} \]

\[ N_{e,j} = \frac{\partial F}{\partial u} \bigg|_{e,j} \times \frac{\partial F}{\partial v} \bigg|_{e,j} \]

\[ = N_{e,j} + \frac{\partial B}{\partial u} \bigg|_{e,j} \left( N_{e,j} \times \frac{\partial F}{\partial v} \bigg|_{e,j} \right) + \frac{\partial B}{\partial v} \bigg|_{e,j} \left( N_{e,j} \times \frac{\partial F}{\partial u} \bigg|_{e,j} \right) \]