Outline

- Classification of projections
  - Geometric vs. non-geometric projections
  - Planar vs. non-planar projections
  - Planar geometric projection
    - Parallel projections
    - Perspective projections

Geometric vs. Non-geometric

- Geometric projection:
  - All projectors are straight lines
- Non-geometric projection:
  - Projectors can be curves
  - Useful for simulating the effect that lights are bended by black holes in space

Planar vs. Non-planar

- Planar projection:
  - The projection surface is plane
- Non-planar projection:
  - Projection surface can be general shapes
  - Cylindrical panorama uses cylinder
  - Circular panorama uses sphere

Planar vs. Non-planar (Cont’d)

- Planar projection:
  - Limited field of view
  - Preserve straight lines
- Non-planar project:
  - Does not preserve straight lines

Planar Geometric Projection

- Parallel
- Perspective
  - Orthographic
  - Elevation
  - Top, Front, & Side
  - Axonometric
  - Isometric, Dimetric, & Trimetric
  - Oblique
  - Cavalier & Cabinet
Parallel Projection
- Different projectors parallel to each other
- Preserve parallelism
- Orthographic:
  - Projectors are perpendicular to the projection plane
- Oblique:
  - Projectors are NOT perpendicular to the projection plane

Orthographic Projection
- Elevation:
  - Projectors are parallel to a principle axis
  - Projection plane is parallel to xy, xz, or yz plane
- Axonometric:
  - Projectors are NOT parallel to any principle axis

Elevations
- Used for:
  - Engineering drawings
  - Architecture drawings
- Advantage:
  - Preserve distance and angle
  - All views are at same scale
- Disadvantage:
  - Hard to understand the 3D shape even all three elevations are given

Matrix Representation for Elevations

Axonometric Projection
- Isometric:
  - Angles between the projector and all 3 principal axes are equal (≈55°)
- Dimetric:
  - Angles between the projector and 2 of the 3 principal axes are equal
- Trimetric:
  - All 3 angles are different

Isometric Projection
- Used for:
  - Patent office records;
  - Furniture design;
  - Video games (SimCity)
- Advantage:
  - Illustrates 3D nature of object
  - Preserve distance along principal axes
- Disadvantage:
  - Do not preserve angle
Oblique Projection
- Essentially an XY-shear transformation
- Preserve shapes along the XY plane

\[ P = \begin{bmatrix} 1 & 0 & \cos \phi & 0 \\ \tan \alpha & 0 & 0 & 0 \\ 0 & 1 & \sin \phi & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

Cavalier & Cabinet Projections
- Oblique projections with special \( \alpha \) angles
  - Cavalier:
    - \( \alpha = 45^\circ \)
    - \( u = x + z \cos(\phi) \)
    - \( v = y + z \sin(\phi) \)
  - Cabinet:
    - \( \alpha = \tan^{-1}2 \approx 63^\circ \)
    - \( u = x + z \cos(\phi)/2 \)
    - \( v = y + z \sin(\phi)/2 \)
    - The \( \phi \) angle is usually 30° or 45°

Perspective Projection
- Different projectors converge to the center of projection
- Does not preserve parallelism
- Foreshortening effect:
  - Objects further from the center of projection appear smaller

Vanishing Point
- Projections of lines that are parallel to a principle axis may converge to a vanishing point
- Based on the number of vanishing points, projective projection is classified into one, two, or three-point projections

One-point & Two-point Projections
- One-point projection:
  - The projection plane is parallel to two of the principle axes
  - One vanishing point
- Two-point projection:
  - The projection plane is parallel to only one of the principle axes
  - Two vanishing points

Matrix Representation
- Configuration:
  - Center of projection is placed at (0,0,0)
  - Projection plane is placed along z = f
  - For 3D point \((x,y,z,1)\):
    - \( w = z / f \)
    - \( u = x / w = x^* f / z \)
    - \( v = y / w = y^* f / z \)

\[
P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix}
\]