Thresholding

Objective:
- Classify pixels based on their intensities
- A simple segmentation approach

Applications:
- Foreground background separation
- Optical character recognition (OCR)

Global Thresholding

Basic idea:
- Set up a global threshold value, which will be used across the image
- Label each pixel based on the comparison between the pixel's intensity and the global threshold

Problems:
- How to pick the threshold?
- Sensitive to global illumination changes

Manual Threshold Selection (Try-and-Error)

Threshold too high
Threshold too low

Histogram-Based Threshold Selection

Involves analyzing the histogram
- Ideally, the histogram is bi-modal
  - Contains 2 narrow peaks & 1 deep valley
  - Threshold should be set at the valley
- For multi-modal histogram, more than 1 threshold may be picked
  - Multilevel thresholding
Otsu’s Method (Cont’d)

Computing Mean Colors

- Naive way of computing $\mu_0$ & $\mu_1$:
  - Go through all pixels in the image
  - Add intensity of each pixel to one of the 2 sums
- More efficient approach:
  - Histogram $h(i)$ stores the number of pixels under each intensity
  - Use them to compute $\mu_0$ & $\mu_1$ directly

Otsu’s Method

- The total variance in image $\sigma^2$ is constant
- Equivalent to maximize between-class variance
  - $\sigma^2_{\text{between}}(t) = \sigma_0^2(t) + \sigma_1^2(t)$
  - $\alpha_0(t)$ & $\mu_0(t)$ can be computed iteratively
  - Maximize $\sigma^2_{\text{between}}(t)$ is more efficient than directly minimizing $\sigma^2_{\text{between}}(t)$

Automatic Threshold Selection

- Select an initial $T$
- Segment image using $T$
  - Produces 2 groups of pixels: $G_0$ & $G_1$
- Compute the average gray value $\mu_0$ & $\mu_1$ for pixels in group $G_0$ & $G_1$
- Compute new threshold:
  - $T' = T \cdot (\mu_0 + \mu_1)/2$
  - Repeat until $|T - T'|$ is small enough

Problem of Global Thresholding

- Proposed by Otsu in 1979
  - Fine a threshold to separate the 2 classes so that their combined spread is minimal
  - Algorithm assumptions:
    - Contains 2 classes of pixels following bimodal histogram
    - Uniform illuminated
    - No need to enforce spatial coherence
  
  Define a weighted within-class variance as:
  - $\sigma^2_{\text{within}}(t) = \omega_0(t)(\bar{x}_0(t) - \mu_0)^2 + \omega_1(t)(\bar{x}_1(t) - \mu_1)^2$
  - $\omega_0$ & $\omega_1$ are the probabilities of the 2 classes separated by threshold $t$
  - $\omega_0(t) = \frac{G_0}{G_0 + G_1}$
  - $\omega_1(t) = \frac{G_1}{G_0 + G_1}$
  - Find the optimal $T$ that:
    - $T = \arg \min T \cdot (\sigma^2_{\text{within}}(t))$

Otsu's Method

- Defined as maximizing the within-class variance
- Find the optimal threshold $T$ that:
  - $\sigma^2_{\text{within}}(t) = \omega_0(t)(\bar{x}_0(t) - \mu_0)^2 + \omega_1(t)(\bar{x}_1(t) - \mu_1)^2$
  - $\omega_0(t)$ & $\omega_1(t)$ are the probabilities of the 2 classes separated by threshold $t$
  - $\omega_0(t) = \frac{G_0}{G_0 + G_1}$
  - $\omega_1(t) = \frac{G_1}{G_0 + G_1}$
  - $T = \arg \min T \cdot (\sigma^2_{\text{within}}(t))$
Adaptive Thresholding

- Basic idea:
  - Select threshold value adaptively based on the intensity values of neighboring pixels
  - Use pixels that are within a \((2W + 1) \times (2W + 1)\) window
  - Eliminate the influence of global illumination changes
  - Different strategies can be used to choose the threshold value

Remove Global Illumination Changes

- Compute a smoothed version of the input image
- Subtract the smoothed version from the original image
- The result gives high frequency information with global illumination changes removed

Mean Approach

- Set the threshold to the mean intensity of local window
- Example:
  - \(7\times7\) window

Mean-C Approach

- Use mean-C, instead of mean, as threshold value
- \(C\) is a constant parameter
- Example:
  - \(7\times7\) window
  - \(C = 7\)
  - Remove the noises on the background

Median-C Approach

- We can also use the median intensity in the local window minus a constant \(C\) as the threshold
- Example:
  - \(7\times7\) window
  - \(C = 4\)
  - The result is not as good as the Mean-C approach