Line Clipping

Outline
- Clipping problem
- Point-in-window test
- Line clipping algorithms
  - Cohen-Sutherland algorithm
  - Parametric algorithm

Clipping Problem
- Remove an object or part of an object that is outside a window
- The object can be:
  - Point, line, or polygon
- The window can be:
  - Rectangle, circle, or arbitrary polygon
- Application:
  - Display part of a large drawing on screen

Point-in-Window Test
- How to define a rectangular window?
  - The coordinates of two corners of the window
    - Bottom left & top right or
    - Top left & Bottom right
- How to determine whether a point is inside a window?
  - \( l \leq x \leq r \)
  - \( b \leq y \leq t \)

Line Clipping
- Both endpoints inside:
  - Accept
- One endpoint inside; one outside:
  - Partially accept
  - Need calculate intersection
- Both endpoints outside:
  - Cannot decide
  - Part of the line may be inside

Cohen-Sutherland Algorithm
- Divide plane into 9 regions, each is assigned a 4-bit code
  - 1st bit: \( y > t \)
  - 2nd bit: \( y < b \)
  - 3rd bit: \( x > r \)
  - 4th bit: \( x < l \)
- Check the codes for the 2 endpoints
- Try to avoid intersection calculations
Trivially Accept
- Both endpoints are in region 0000
  - Both are in the window
  - Whole line is inside
  - Trivially accept

Trivially Reject
- Codes for the two endpoints are 1001 & 1010
  - 1001 AND 1010 = 1000
  - Both endpoints are above the window
  - Trivially reject
  - Cannot reject if the AND operation outputs 0000

Partially Accept
- 0001 AND 0000 = 0000
  - Cannot decide
- 0001 XOR 0000 = 0001
  - Intersect with left edge
- Intersection with left splits it into 2 parts
  - Left part has code 0001 & 0001
    - Reject
  - Right part has code 0000 & 0000
    - Accept

Non-trivial Reject
- 1000 AND 0010 = 0000
  - Cannot decide
- 1000 XOR 0010 = 1010
  - Intersect with top & right edges
- Intersection with top splits it into 2 parts
  - Top part has codes 1000 & 1010
    - Reject
  - Bottom part has code 0010 & 0010
    - Reject

Recursive Case
- 1000 AND 0101 = 0000
  - Cannot decide
- 1000 XOR 0101 = 1101
  - Intersect with top, bottom, & left edges
- Intersection with top splits it into 2 parts
  - Top part are rejected
  - Bottom part has codes 0000 & 0101
    - Cannot decide
    - Do it again

Pseudocode for Cohen-Sutherland Algorithm
```c
void LineClipping(int endpoint1, int endpoint2) {
    if (endpoint1==0000 && endpoint2==0000) {
        // Accept and draw the current line segment;
        return;
    }
    if (endpoint1 & endpoint2 != 0000) {
        // Reject the current line segment;
        return;
    }
    // Compute endpoint1 XOR endpoint2;
    // Select a boundary for intersection calculation;
    // intersect1 = code at one side of boundary;
    // intersect2 = code at the other side of boundary;
    LineClipping(endpoint1, intersect1);
    LineClipping(intersect2, endpoint2);
}
```
**Parametric Algorithm**

- Use the parametric line function:
  - \( x = x_0 + k(x_1 - x_0) \)
  - \( y = y_0 + k(y_1 - y_0) \)
- Find the range of \( k \) that is inside the window:
  - \( k_{in} \leq k \leq k_{out} \)
- No need for recursion, but always requires 4 intersection calculations

**Intersection Calculation**

- 4 intersections exist in general cases
- At intersections:
  - \( l = x_0 + k(x_1 - x_0) \)
  - \( r = x_0 + k(x_1 - x_0) \)
  - \( t = y_0 + k(y_1 - y_0) \)
  - \( b = y_0 + k(y_1 - y_0) \)
- Therefore:
  - \( k_1 = (t - x_0)/(x_1 - x_0) \)
  - \( k_2 = (r - x_0)/(x_1 - x_0) \)
  - \( k_3 = (t - y_0)/(y_1 - y_0) \)
  - \( k_4 = (b - y_0)/(y_1 - y_0) \)

**Potential Entering/Leaving Point**

- 2 intersections for the horizontal zone
  - One is horizontal entering point
    - \( H_{in} = \min(k_1, k_2) \)
  - The other is horizontal leaving point
    - \( H_{out} = \max(k_1, k_2) \)
- 2 intersections for the vertical zone
  - \( V_{in} = \min(k_3, k_4) \)
  - \( V_{out} = \max(k_3, k_4) \)

**Real Entering/Leaving Point**

- The real entering point:
  - Should be the larger of the two entering points
  - Should also \( \geq 0 \)
  - \( k_{in} = \max(0, H_{in}, V_{in}) \)
- Similarly, the real leaving point:
  - \( k_{out} = \min(1, H_{out}, V_{out}) \)

**Outside Cases**

- If \( k_{in} > k_{out} \)
  - The line segment leaves the window before it enters
  - Whole line segment is outside the window
  - Fully reject

**Inside Portion**

- If \( k_{in} < k_{out} \)
  - Range [\( k_{in}, k_{out} \)] is inside the window
  - The coordinates of endpoints of the inside portion can be calculated using:
    - \( x_s = x_0 + k_{in}(x_1 - x_0) \)
    - \( y_s = y_0 + k_{in}(y_1 - y_0) \)
    - \( x_e = x_0 + k_{out}(x_1 - x_0) \)
    - \( y_e = y_0 + k_{out}(y_1 - y_0) \)
Special Cases

- How about horizontal/vertical lines?
  - Edges parallel to the line can be ignored
- Two situations:
  - Line is outside the window
  - The inside portion can be calculated using two intersections