Sharpening Filter

Outline
- What is sharpening
- Derivatives of digital functions
- Derivative filters:
  - Prewit filter
  - Sobel filter
  - Laplacian filter
- Sharpening filters:
  - Laplacian sharpening
  - High-boost filtering
  - Unsharp masking

Objectives:
- Highlight fine details in an image
- Enhance details that have been blurred
- Reduce blurriness in misfocused photos

1st Order Derivatives
- Properties:
  - Zero in flat segments;
  - Nonzero at the onset of a step or ramp
  - Nonzero along a ramp of constant slope

\[ \frac{\partial f}{\partial x} = f(x+1) - f(x) = f(x) - f(x-1) \]

Example of Derivatives Calculation

2nd Order Derivatives
- Properties:
  - Zero in flat segments;
  - Nonzero at the onset and end of a step or ramp
  - Zero along a ramp of constant slope

\[ \frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial f}{\partial x} = f(x+1) + f(x-1) - 2f(x) \]
Image Gradient

- Calculates the 1st order derivatives
- The gradient of an image is a vector field
- Gradient direction is the direction of most rapid change in intensity
- Gradient magnitude gives the edge strength

\[ \nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \]

\[ \theta(\nabla f) = \tan^{-1}\left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right) \]

\[ |\nabla f| = \sqrt{\left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2} \]

\[ \nabla f[x,y] = f[x+1,y] - f[x,y] \]

\[ \nabla f[x,y] = f[x,y+1] - f[x,y] \]

Prewit Filter

- Both \( D_x \) and \( D_y \) are separable:
- Apply mean filter in one direction and take derivative in the other direction

\[
D_{\text{Prewit}} = \frac{1}{3} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & -1 & 0 & 1 \\ -1 & 0 & 1 & 1 \end{bmatrix}
\]

Sobel Filter

- Both \( D_x \) and \( D_y \) are separable:
- Apply triangle filter in one direction and take derivative in the other direction

\[
D_{\text{Sobel}} = \frac{1}{4} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & -1 & 0 & 1 \\ -1 & 0 & 1 & 1 \end{bmatrix}
\]

\[
D_{\text{Sobel}} = \frac{1}{4} \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}
\]

Result of Sobel Filter

Original: X-dir, Y-dir, Sobel sharpening

Image Laplacian

- Image Laplacian calculates the 2nd order derivatives
- The Laplacian of an image is a scalar field
- Digital image Laplacian is calculated using the color difference between nearby pixels and the center pixel

\[
\Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}
\]

\[
\nabla^2 f[x,y] = -4 \times f[x,y] + f[x+1,y] + f[x-1,y] + f[x,y+1] + f[x,y-1]
\]
**Laplacian Filter**

- Image Laplacian can be calculated using one of the Laplacian filters
  - The number of neighbors involved in the filter can be either 4 or 8
  - The center coefficient of the filter can be either positive or negative

\[
\begin{bmatrix}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0
\end{bmatrix}
\quad
\begin{bmatrix}
0 & -1 & 0 \\
-1 & 4 & -1 \\
0 & -1 & 0
\end{bmatrix}
\]

**Laplacian Sharpening**

- Obtain sharpening results by subtracting (or adding) the image Laplacian from the original image
  - Naïve implementation requires two rendering passes
  - Can be implemented in one pass using a modified filter

\[
f_{ls}(x,y) = f(x,y) - \Delta f(x,y)
\]

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\quad
\begin{bmatrix}
0 & 1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

**Result of Laplacian Filter**

- Original
  - 4 neighbors
  - 8 neighbors

**Result of High-boost Filtering**

- Nullify the darkening effect of high-pass filter by adding original image's intensity
  - Parameter A controls the contribution:
    - A=1: Laplacian filter
    - A=2: Laplacian sharpening
    - 1<A<2: Limited low frequency
    - A>2: Brightened image with edge enhanced

\[
f_{hb}(x,y) = A \cdot f(x,y) - f_{ls}(x,y)
\]

\[
f_{ls}(x,y) = (A-1) \cdot f(x,y) + f_{ls}(x,y)
\]

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\quad
\begin{bmatrix}
1 & 1 & 1 \\
0 & 9 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 9 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]

**Unsharp Masking**

- Originates from darkroom photography:
  - Clamp a blurred positive to the original negative
  - Blurred positive cancels the low frequency signal
  - Develop the combined negative on contrasty photographic paper
  - Partial cancellation emphasizes the high frequency edges

- For digital processing:
  - Step 1: detect edges and create unsharp mask:
  - Subtract a smoothed version of an image from the image itself
  - Step 2: Increase contrast at edges
  - Selectively increase contrast along edges using unsharp mask
  - Can be done in one pass
Unsharp Masking Process

Blur → Subtract → Mask → Increase contrast