Sharpening Filter

Outline

- What is sharpening
- Derivatives of digital functions
- Derivative filters:
  - Prewit filter
  - Sobel filter
  - Laplacian filter
- Sharpening filters:
  - Laplacian sharpening
  - High-boost filtering
  - Unsharp masking

Sharpening

Objectives:

- Highlight fine details in an image
- Enhance details that have been blurred
- Reduce blurriness in misfocused photos

1st Order Derivatives

- Properties:
  - Zero in flat segments;
  - Nonzero at the onset of a step or ramp
  - Nonzero along a ramp
- Discrete form:
  \[ \frac{df}{dx} = f(x + 1) - f(x) \]
  \[ f(x) - f(x - 1) \]

2nd Order Derivatives

- Properties:
  - Zero in flat segments;
  - Nonzero at the onset & end of a step or ramp
  - Zero along a ramp of constant slope
- Discrete form:
  \[ \frac{d^2f}{dx^2} = \frac{f(x + 1) - f(x)}{dx} - \frac{f(x) - f(x - 1)}{dx} \]
  \[ f(x + 1) + f(x - 1) - 2f(x) \]

Example of Derivatives Calculation

Step Ramp

1st order derivative

2nd order derivative
Image Gradient

- Defined using 1st order derivative
- The gradient of a 2D scalar function is a vector field
  \[ \nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \]
- Gradient direction is the direction of most rapid change in intensity
  \[ \theta(\nabla f) = \tan^{-1} \left( \frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}} \right) \]
- Gradient magnitude gives the edge strength
  \[ \| \nabla f \| = \sqrt{ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 } \]
- Digital image gradient is calculated by nearby pixels' color difference

\[ \nabla F(x, y) = F(x+1, y) - F(x, y) \]
\[ \nabla F(x, y) = F(x, y+1) - F(x, y) \]

Prewit Filter

- A linear filter for image gradient calculation
- Apply mean filter in one direction and take derivative in the other direction

\[ D_{\text{Prewit}} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \]

\[ D_{\text{Prewit}} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \]

Both \( D_{\text{Prewit}} \) & \( D_{\text{Prewit}}^T \) are separable

Sobel Filter

- Another linear filter for gradient calculation
- Apply triangle filter in one direction and take derivative in the other direction

\[ D_{\text{Sobel}} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \]

\[ D_{\text{Sobel}} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix} \]

Sobel sharpening:
- Enhance edges by adding image gradient to the original image

Image Laplacian

- Defined using 2nd order derivatives
- The Laplacian of a 2D scalar function is a scalar field
  \[ \Delta f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \]
- Digital image Laplacian is calculated using the color difference between nearby pixels and the center pixel

\[ \Delta F(x, y) = F(x+1, y) + F(x-1, y) + F(x, y+1) + F(x, y-1) - 4F(x, y) \]

Laplacian Filter

- Image Laplacian is a linear operation and can be calculated using one of the Laplacian filters
- The sign of coefficients can be negated

\[ \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \]

By definition, only 4 neighboring pixels are involved, 8-neighbors version is also used

\[ \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} \]
**Laplacian Sharpening**

- Sharpen an image by subtracting (or adding) the image Laplacian from the original image
  \[ f_{LS}(x, y) = f(x, y) - \Delta f(x, y) \]
- Naïve implementation requires 2 passes
  - First compute image Laplacian, followed by sharpening results
- Can be implemented in 1 pass using a modified filter
  \[
  \begin{pmatrix}
  0 & 0 & 0 & 0 \\
  0 & 1 & 0 & -1 \\
  0 & 0 & 1 & 0
  \end{pmatrix}
  \]

**Result of Laplacian Filter**

**High-boost Filtering**

- Amplify an input image, then subtract from it a blurred version of the image
  \[ f_{HB}(x, y) = A \cdot f(x, y) - f_{LS}(x, y) \]
- Parameter A controls the contribution:
  - \( A = 1 \): Laplacian filter
  - \( 1 < A < 2 \): Limited low frequency
  - \( A = 2 \): Laplacian sharpening
  - \( A > 2 \): Brightened image with edge enhanced

**Result of High-boost Filtering**

**Unsharp Masking**

- Originates from darkroom photography:
  - Clamp a blurred positive to the original negative
  - Blurred positive cancels the low frequency signal
  - Develop the combined negative on contrasty photographic paper
  - Partial cancellation emphasizes the high frequency edges
- For digital processing:
  - Step 1: detect edges and create unsharp mask:
    - Subtract a smoothed version of an image from the image itself
  - Step 2: Increase contrast at edges
    - Selectively increase contrast along edges using unsharp mask
  - Can be done in one pass

**Unsharp Masking Process**