Investigations of Wilson's and Traulsen's Group Selection Models in Evolutionary Computation

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Abstract. Evolving cooperation by evolutionary algorithms is impossible without introducing extra mechanisms. Group selection theory in biology is a good candidate as it explains the evolution of cooperation in nature. Two biological models, Wilson's trait group selection model and Traulsen's group selection model are investigated and compared in evolutionary computation. Three evolutionary algorithms were designed and tested on an *n-player* prisoner's dilemma problem; two EAs implement the original Wilson and Traulsen models respectively, and one EA extends Traulsen's model. Experimental results show that the latter model introduces high between-group variance, leading to more robustness than the other two in response to parameter changes such as group size, the fraction of cooperators and selection pressure.

Key words: the evolution of cooperation, evolutionary computation, Wilson's trait group selection model, Traulsen's group selection model

1 Introduction

Evolutionary computation (EC) is often viewed as an optimization process, as it draws inspiration from the Darwinian principle of variation and natural selection. This implies that EC may fail to solve problems which require a set of cooperative individuals to jointly perform a computational task. When cooperating, individuals may contribute differently, and hence might lead to unequal fitnesses. Individuals with lower fitness will be gradually eliminated from the population, despite their unique contributions to overall performance of the algorithm. Hence, special mechanisms should be implemented in EC that avoid selecting against such individuals.

In nature, the success of cooperation is witnessed at all levels of biological organization. A growing number of biologists have come to believe that the theory of group selection is the explanation even though this theory has been unpopular for the past 40 years; hence new models and their applications are investigated

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[2]. Individuals are divided into groups, and only interact with members in the same group. The emergence of cooperation is due to competition between individuals and between groups. Individual competition selects against individuals with lower fitness, but group competition favors individuals who cooperate with others, regardless to their individual fitness. The group selection model proposed by Wilson and Sober [9,12] and the model by Traulsen and Nowak [10] represent two research strands in this area; groups in Wilson's model are mixed periodically during evolution, while groups in Traulsen's model are isolated. Hence, selection between groups and within groups work differently in these models.

Extending these two models to encourage cooperation in artificial evolution is a relatively new research direction; most research [1,3,6,7] so far is based on Wilson's model or its variations, not on Traulsen's. This motivated us to investigate the role each model can play to encourage cooperation in EC, and to analyze their differences. Three evolutionary algorithms adapting the two models were designed and examined under different parameter settings; these parameters refer to group size, fraction of cooperators and selection pressure, and they directly affect the selection dynamics. Our results show that the algorithm which extends Traulsen's model is more robust towards parameter changes than the algorithms implementing the original Wilson and Traulsen models.

The reminder of this paper is organized as follows. Section 2 introduces the three evolutionary algorithms. Section 3 describes the experiments and the results obtained. Section 4 concludes.

2 Algorithms Design

Wilson's and Traulsen's models interpret the idea of group selection in a different fashion. In Wilson's model, groups reproduce proportional to group fitness. Offspring groups are periodically mixed in a migrating pool for another round of group formation. The number of individuals a group contributed to this pool is proportional to its size; so cooperative groups contribute more to the next generation. On the contrary, Traulsen's model keeps groups isolated. An individual is selected proportional to its fitness from the entire population, and the offspring is added to its parent's group. When the group reaches its maximal size, either an individual in this group is removed, or the group splits into two, so another group has to be removed. Cooperative groups grow faster, and therefore, split more often. For detailed descriptions of these models, please refer to [9,10,12].

Our study aims to investigate the performance of the two models in extending evolutionary algorithms to evolve cooperation. The investigation is conducted in the context of the *n-player* prisoner's dilemma (NPD). The NPD game offers a straightforward way of thinking about the tension between the individual and group level selection [4]; meanwhile it represents many cooperative situations in which fitness depends on both individual and group behavior. In this game, N individuals are randomly divided into m groups. Individuals in a group independently choose to be a cooperator or a defector without knowing the choice of others. The fitness function of cooperators ($f_{C_i}(x)$) and defectors ($f_{D_i}(x)$) in group i are specified by the following equations:

$$f_{C_i}(x) = base + w(\frac{b(n_i q_i - 1)}{n_i - 1} - c), \quad (0 \le i < m) \tag{1a}$$

$$f_{D_i}(x) = base + w\frac{b n_i q_i}{n_i - 1}, \quad (0 \le i < m) \tag{1b}$$

where $base$ is the base fitness of cooperators and defectors; q_i the fraction of cooperators in group i; n_i the size of group i; b and c are the benefit and cost caused by the altruistic act, respectively; w is a coefficient. Evidently, cooperators have a lower fitness than defectors, because they not only pay a direct cost, but also receive benefits from fewer cooperators than defectors do. The fitness of group i is defined as the average individual fitness. Although defectors dominate cooperators inside a group, groups with more cooperators have a higher group fitness. Hence, the dynamics between individual and group selection will drive the game in different directions.

A simple evolutionary algorithm implementing Wilson's model (denoted as W) is described in Algorithm 1. This algorithm starts with randomly initializing

Algorithm 1: An Evolutionary Algorithm Based on Wilson's Model

```
P ← Initialize_Population(N, r);
while population does not converge or max generation is not reached do
    P' ← Disperse_Population(P, m);
    Evaluate_Fitness(P');
    for i ← 0 to N' do
        gn ← Select_Group(P');
        idv ← Select_Individual(P', gn);
        idv' ← Reproduce_Offspring(idv);
        Add_Individual(idv', NP, gn)
    end
    P ← Mixing_Proportionally(NP);
end
```

a population P with N individuals, r percent of which are cooperators. P is then divided into m groups, and the individual and group fitness of the dispersed population P' is evaluated. Afterwards, reproduction begins; a group with number gn is first selected, from which an individual idv is selected to produce offspring idv'. idv' is then added to group gn in the new population NP. The reproduction causes groups in NP vary in size, because the selection of groups is proportional to fitness. In total N' offspring will be produced, where N' is normally larger than population size N. This gives cooperators an opportunity to increase their frequency in the next generation. To maintain the original population size N, groups in NP are mixed and each contributes individuals proportional to its size to new population P. P will repeat the above steps until the population converges or the maximum number of generations is reached.

Similarly, Traulsen's is embedded into an evolutionary algorithm shown in Algorithm 2. This algorithm initializes, divides, and evaluates the population

Algorithm 2: An Evolutionary Algorithm Based on Traulsen's Model

```
P ← Initialize_Population(N, r);
P′ ← Disperse_Population(P, m);
while population does not converge or max generation is not reached do
    Evaluate_Fitness(P′);
    for i ← 0 to N″ do
        idv ← Select_Individual_from_Population(P′);
        idv′ ← Reproduce_Offspring(idv);
        Put_Back_to_Group(idv′, gn);
        if Group_Size(gn) > n then
            rnum ← Generate_Random_Number(0, 1);
            if rnum < q then
                Split_Group(gn);
                Remove_a_Group();
            else
                Remove_an_Individual_in_Group(gn);
            end
        end
    end
end
```

the same way algorithm W does. However, there are two major differences. First, the population only disperses once at the beginning of the process; the groups are kept isolated afterwards. Second, the reproduction step is different. An individual idv is selected from the entire population for reproduction, rather than from a group. Offspring idv' is put back into its parent's group, group gn. If the size of group gn exceeds the pre-defined group size n, a random number $rnum$ is generated. If $rnum$ is less than a group splitting probability q, group gn splits and its individuals are randomly distributed into offspring groups. A group has to be removed to maintain a constant number of groups; otherwise, an individual from group gn is removed. In Traulsen's model, a group or an individual to be eliminated is randomly selected. As an extension, we also investigate selecting such a group or individual reversely proportional to its fitness. Therefore, two variations of Algorithm 2 are implemented, one refers to the former (denoted as T1) and the other to the latter (denoted as T2).

3 Investigations with the Algorithms

The investigations focus on the effects caused by different group size n, initial fraction of cooperators r, and coefficient w. Parameters n and r affect the assortment between cooperators and defectors in groups, and coefficient w affects the individual and group fitness; both cause changes in selection dynamics.

To focus on the selection dynamics, we assume asexual reproduction without the interference of mutation. A roulette wheel selection is adopted in the reproduction step for all algorithms. Parameters that are common to all experiments are set as follows: runs $R = 20$, generation $gen = 5,000$, population size

$N = 200$, base fitness $base = 10$, benefit $b = 5$, cost $c = 1$, group splitting probability $q = 0.05$, N''=10, and N' is decided by the following equation [9].

$$N' = \sum_{i=1}^{m} n_i \times (q_i \times f_{C_i}(x) + (1 - q_i) \times f_{D_i}(x)) \quad (2)$$

For each algorithm, we measure the success ratio by the number of runs whose population converges to cooperators to the number of total runs 20. The larger the ratio, the more likely an algorithm favors cooperation. We also collect the average variance ratio [5], which indicates composition difference between groups. The higher this ratio, the more prominent the effect of group selection.

3.1 The effects of group size and initial fraction of cooperators

First we investigate how the three algorithms behave under different group sizes. We set $r = 0.5$ and $w = 1$. Group size n is varied from $\{5, 10, 20, 50, 100\}$. The success ratio and average variance ratio (in brackets) for each setting are listed in the first 3 columns in Table 1. As can be seen, the performance of T1 degrades

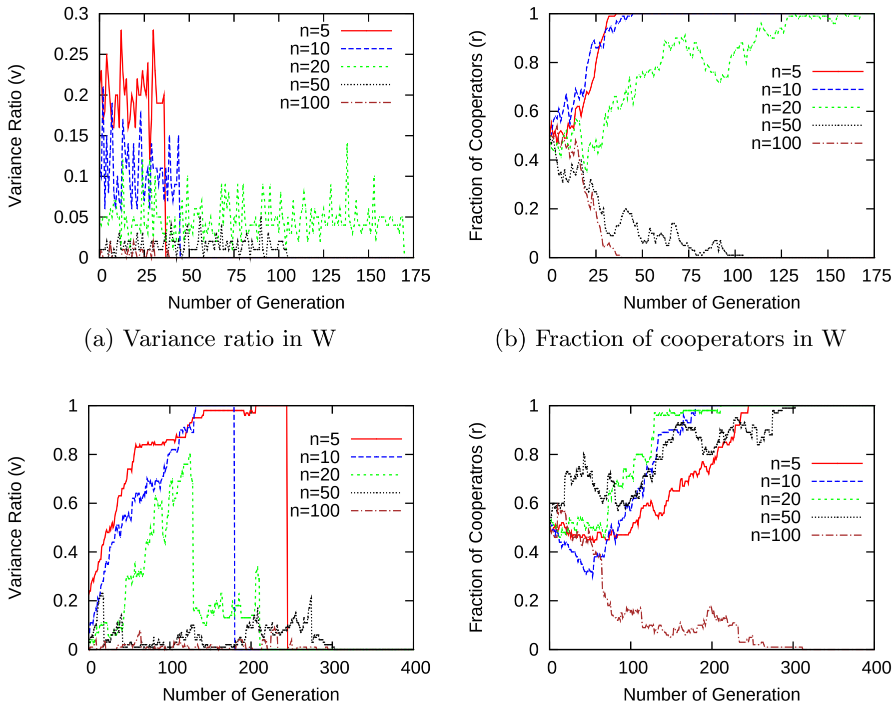
	r=0.5			r=0.3			r=0.1		
n	W	T2	T1	W	T2	T1	W	T2	T1
5	1(0.196)	1(0.820)	1	1(0.201)	1(0.853)	0.95	1(0.196)	1(0.893)	0.7
10	1(0.092)	1(0.655)	0.85	1(0.098)	1(0.665)	0.55	1(0.095)	1(0.767)	0.2
20	0.8(0.045)	1(0.291)	0.65	0.55(0.045)	1(0.398)	0.25	0.25(0.042)	0.65(0.465)	0.1
50	0(0.015)	1(0.112)	0.15	0(0.016)	0.8(0.105)	0.1	0(0.015)	0.55(0.049)	0.05
100	0(0.004)	0(0.011)	0	0(0.005)	0(0.014)	0	0(0.005)	0(0.015)	0

Table 1: The effects of group size n and initial fraction of cooperators r on the three algorithms.

as n grows. The population in W converges to cooperators when small groups are employed ($n = 5$ or 10). As n increases, evolving cooperation becomes difficult. In contrast, T2 converges to cooperators except $n = 100$.

The observation can be explained by Figure 1. Figure 1(a) shows that variance ratio v in W decreases as n increases, which diminishes the effect of group selection. As a result, the selection on the individual level is becoming the dominate force, so the population converges quicker to defectors, see Figure 1(b). The same trend between v and n is also observed in T2. However, given n ranges from 5 to 50, its v value is much higher than or equal to the highest v value of W (see Figure 1(c)). This implies that T2 preserves variance between groups better than W, and explains why T2 is more effective than W in evolving cooperation. Unlike W, the convergence speed to cooperators of T2 is not accelerated as n gets smaller; for example, runs with $n = 10$ converge first. When groups are too small or too large, much averaging is required to remove defectors from the population (see Figure 1(d)).

We further adjusted the value of r from 0.5 to 0.3 and 0.1. We were curious about the response of the three algorithms to this change, because when r drops,



(a) Variance ratio in W

(b) Fraction of cooperators in W

(c) Variance ratio in T2

(d) Fraction of cooperators in T2

Fig. 1: The variance ratio v and fraction of cooperators r for algorithm W and T2 under different group sizes when $r = 0.5$ and $w = 1$.

the number of cooperators assigned to groups is smaller, which increases the influence of individual selection in a group. As shown in Table 1, the performance of T1 decreases as r drops. For W and T2, when n is small (5 or 10), due to the strong group selection effects, the decrease of r does not affect the success ratio, but only slows convergence speed towards cooperation; for larger groups, as n increases (group selection is weaker) and r decreases (individual selection is stronger), group selection can hardly dominate individual selection; so it becomes difficult for both algorithms to preserve cooperation. However, T2 is less affected, because for a given group size, similar v values in W are observed despite the changes of r, while relatively high v values are produced by T2 even r drops.

3.2 Weak *vs.* strong selection

The composition of groups is not the only factor that drives selection dynamics; a difference in fitness values of cooperators and defectors is another one. It affects the pressure put on groups and individuals. In the next experiment, we use coefficient w to adjust the selection pressure. If w is small, the selection is called weak selection; otherwise it is called strong selection.

We tested the three algorithms with r=0.5 and w set to {0.1, 0.5, 1, 2, 5, 10}, respectively on all group sizes. Results are shown in Table 2. One first

	w=0.1			w=0.5			w=1		
n	W	T2	T1	W	T2	T1	W	T2	T1
5	1(0.197)	1(0.949)	0.6	1(0.201)	1(0.884)	0.9	1(0.196)	1(0.820)	1
10	1(0.095)	1(0.766)	0.6	1(0.096)	1(0.515)	0.8	1(0.092)	1(0.655)	0.85
20	0.85(0.044)	0.95(0.601)	0.55	1(0.046)	1(0.370)	0.65	0.8(0.045)	1(0.291)	0.65
50	0.4(0.015)	0.45(0.174)	0	0(0.015)	1(0.115)	0.3	0(0.015)	1(0.112)	0.15
	w=2			w=5			w=10		
5	1(0.196)	1(0.806)	1	0.9(0.196)	1(0.820)	1	0(0.190)	1(0.875)	1
10	1(0.096)	1(0.543)	0.9	0.1(0.096)	1(0.596)	0.8	0(0.096)	1(0.638)	0.85
20	0.1(0.042)	1(0.309)	0.8	0(0.050)	0.8(0.334)	0.5	0(0.046)	0.8(0.347)	0.15
50	0(0.014)	0.8(0.079)	0.15	0(0.014)	0.45(0.021)	0	0(0.016)	0.1(0.053)	0

Table 2: The performance of the algorithms under weak and strong selection.

notices that the performance of the three algorithms increases and then deceases as selection pressure goes from weak to strong. If selection is too weak, the fitness between two roles and between groups are very close. Hence, group and individual selection become neutral, especially if large groups are adopted, so defectors can more easily take over the population. If the selection is too strong, though group selection still favors cooperative groups, because the larger fitness difference between both roles, cooperators are more difficult to be selected. To be more specific, for small groups (n = 5 or 10) only T2 can successfully preserve cooperation under both weak and strong selection. The increase of selection pressure raises the influence of individual selection. In response to this increase, the variance ratio in W for a given group size does not change at all, while T2 still keeps noticeable high variance ratios. This also explains why T2 outperforms W with larger groups.

3.3 Discussion

The above experiments demonstrate that maintaining variance between groups has great impact on group selection models. For W, if groups are randomly formed, small group sizes are desired because small groups increase group variance. This confirms previous investigations (see [5,8,11] for examples). We further show that such a requirement only works if the selection is weak. T2, because it is able to introduce high group variance, is more robust towards parameter changes. The reason lies in how the two models manage groups. Mixing and reforming of groups in Wilson's model constantly averages the variance between groups, so in Figure 1(a) we observe the variance between groups fluctuating. In contrast, because groups in Traulsen's model are kept isolated, and the selection step in reproduction is proportional to individual fitness, the fraction of cooperators in a cooperative group grows faster than in a less cooperative group, hence gradually increases the variance between groups. T2 performs better than T1 under all settings, because removing an individual or a group according to reversed fitness value at death selection is very likely wiping out defectors, thus it certainly helps cooperators.

4 Conclusion

Wilson's and Traulsen's models are possible extensions of EC to evolve cooperation. Here, we investigated evolutionary algorithms that adapt the two models, and analyzed their differences. Our experimental results show that an algorithm which extends Traulsen's model is less sensitive to parameter changes than the algorithms based on the original Wilson and Traulsen models, because it is able to maintain high between-group variance, which is able to override individual selection arisen by the parameter changes. Future work will consider theoretically investigate the extended algorithm; its extensions to multilevel selection; and its role in the theory of evolutionary transition.

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