

# Self-organizing Systems

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## Glossary

**Attractor** A special set of system states approached by a dynamical system after some time has passed when starting from a variety of initial states.

**Autopoiesis** The process by which systems maintain their identity and organization and regenerate their components in the course of their operation.

**Competition and Cooperation** Types of interaction between two or more elements of a system. Competition refers to each element striving to maximize its use of a finite and/or non-renewable resource. Cooperation refers to the elements engaging in a mutually beneficial exchange.

**Complexity** Measure of number of elements and way of their interaction (structural c.); measure of variety of behavioral repertoire of a system (functional c.).

**Constructive system** A system whose later components are generated

during the interaction of its earlier components.

**Dynamics** The quantitative development of a system's state variables over time.

**Emergence** The appearance of qualitatively new phenomena on higher levels of a hierarchical system.

**Evolution** A process of structural or qualitative change in some direction.

**Instability** Inability of a system to keep its state or structure.

**Mode** Macroscopic behavior of a system caused by the interaction of its microscopic parts via long-range correlations.

**Non-equilibrium** System state with inflow of matter, energy and/or information causing it to stay away from its most probable state under the hypothetical condition of isolation.

**Phase Transition** A point at which the appearance or behaviour, or qualitative nature of the steady state of a system changes suddenly.

**Resilience** Measure of a system's ability to remain within a domain of stability in response to fluctuations of the system by a perturbation, and the ability of the system to return to that stable domain having once left.

**Self-organized Criticality** The ability of a system to evolve in such a way as to approach a critical point and then maintain itself at that point.

## I Definition of Subject and its Importance

*Self-organization* is a core concept of Systems Science. It refers to the ability of a class of systems (self-organizing systems (SOS)) to change their internal structure and/or their function in response to external circumstances. Elements of self-organizing systems are able to manipulate or organize other elements of the same system in a way that stabilizes either structure or function of the whole against external fluctuations. The process of self-organization is often achieved by growing the internal space-time complexity of a system and results in layered or hierarchical structures or behaviors. This process is understood not to be instructed from outside the system and is therefore called *self-organized*.

Modern ideas about self-organization start with the foundation of cybernetics in the 1940s. W. Ross Ashby, H. von Foerster and N. Wiener, among others, have contributed to an early understanding. Later, the concept was adopted in physics

and nowadays pervades most of natural sciences. Many systems have been identified as possessing aspects of self-organization, though a clear definition is still lacking. As a result of this inaccuracy, the theory of self-organization is still in its infancy. While the concept has found applications in the social sciences and engineering as well, SOSs are an area of active research, with fundamental questions still being explored.

## II Introduction

Over the last decades a variety of features have been identified as typical for self-organizing systems. Not all of these features are present in all systems able to self-organize. Self-organizing systems are *dynamic*, often *non-deterministic*, *open*, exist *far from equilibrium* and sometimes employ *autocatalytic amplification of fluctuations*. Often, they are characterized by *multiple time-scales* of their internal and/or external interactions, they possess a *hierarchy of structural and/or functional levels* and they are able to *react* to external input *in a variety of ways*. Many self-organizing systems are *non-teleological*, i.e. they do not have a specific purpose except their own existence. As a consequence, *self-maintenance* is an important function of many self-organizing systems. Most of these systems are *complex* and use *redundancy* to achieve *resilience* against external perturbation tendencies.

Key aspects of self-organizing systems are:

- Growth of Complexity
- Emergence of new phenomena
- Positive and negative feedback loops of internal regulation

The process of self-organization has been invoked to explain numerous phenomena in the natural sciences. From non-living systems like galaxies and stars down to nanoparticle aggregates, self-organizing systems have been observed. In the living world cells, organisms and ecosystems provide examples of systems classified as self-organizing. The concept has found applications in man-made systems like communication networks, societies, economies, and has been identified to be at work in the world of ideas in the development of world views, scientific beliefs and norm systems.

## III History of the concept of self-organization

### III.1 Early History

The concept of self-organization can be traced back to at least two sources: Western philosophy influenced heavily by Greek thinking; and eastern philosophy, centered around the process thinking of Bhuddism. The ideas derived from both sources resound with the modern way of thinking about self-organization although the word itself had never been used.

On wondering about the origin of the world, Greek atomists from Democritus of Abdera to Epicurus of Samos argued that world order arose from chance collisions of particles. First, the cosmos (from Greek *kosmos = the ordered*) did not exist but chaos instead (from Greek *chaos = the disordered*). In modern times chaos theory has taken up this topic again, with deep connections to ideas about self-organization and the origin of order in the universe.

In the Christian tradition, St Thomas Aquinas contributed through his interest in logical proofs for the existence of God. One of these proofs considered God to be the ultimate organizer or designer. The argument was that everything had to be organized and this called for an organizer. In turn, the organizer had to be organized and so on back to the original organizer: this was God. Since God is present without cause (otherwise he would have to be organized by another entity), he must have somehow organized himself.

The Bhuddist way of thinking, on the other hand, was fundamentally process-oriented. Things are considered not to be in static existence, but rather are thought to be generated and maintained by proper processes. The emphasis on processes is reminiscent of self-organizing systems whose structure is determined by proper processes of internal and external interactions.

### III.2 The first use of the term

Work on General Systems Theory (von Bertalanffy) [1] and Cybernetics (Wiener) [2] paved the way for the idea of self-organization.

The concept of a self-organizing system was introduced by Ashby in 1947 [3]. In the 1950s a self-organizing system was considered to be a system which changes its basic structure as a function of its experience and environment. The term was used by Farley and Clark in 1954 to describe learning and adaptation mechanisms [4]. Ashby [5], in 1960, redefined a self-organizing system to include the environment with the system proper. Von Foerster argued [6], also in 1960, that a self-organizing dynamical system possesses some stable structures (eigenvalues, attractor states) which he later termed eigenbehavior.

### III.3 Further developments

This notion was further developed by Haken [7] in 1977 who termed the global cooperation of elements of a dynamical system - resulting in it assuming an attractor state - *self-organization*. Both Haken and Kauffman (1993) [41] argued for a deep connection between self-organization and *selection*. Haken found that modes of collective behaviour are competing against each other and considered this process to be Darwinian selection in the non-living world. Kauffman, on the other hand, emphasized the role of constraints on the direction of evolution (mostly of the living), caused by self-organization.

Already in the 1970s, however, ideas branched out into different directions. One branch of the development of the idea deepened the relation to studies of learning and adaptation (Conrad, Kohonen, [9, 10]), another branch studied processes of self-organization in systems far from equilibrium (Prigogine, Haken)

[11, 12]. Chaos theory (Thom, Mandelbrot) [13, 14] was the line of inquiry into nonlinear systems in mathematics, whereas autopoiesis and self-maintenance where at center stage in biology (Eigen, Rosen) [15, 16] neurophysiology (von der Mahlsburg, Linsker [17, 18]) and cognitive science (von Foerster, Maturana and Varela) [19, 20].

In recent years, self-organizing systems have assumed center stage in the natural sciences [21, 22], and the social sciences [23, 24, ?]. Engineering is beginning to see the usability of the concept [26] in connection with the approach of nano-scale applications and the growing complexity of human artefacts.

## IV Examples of natural self-organizing systems

Classical examples of natural self-organizing systems are the formation of Be-nard convection cells in non-equilibrium thermodynamics, the generation of laser light in non-linear optics and the Belousov-Zhabotinsky reaction in chemistry. These are examples from the non-living world, and the complexity of resulting macroscopic space-time patterns is restricted.

Nearly unrestricted complexity through self-organization can be achieved in the living world. For instance, the interaction of species in foodwebs could be looked at from this point of view [22]. Here, we shall briefly look at the self-organization of the Earth's biosphere known as the Gaia hypothesis [27]. This hypothesis states that Earth's living and non-living components self-organize into a single entity called *Gaia*. Gaia can be understood as the whole of the biosphere, that is able to self-stabilize. The model states, in other words, that the biomass of Earth self-regulates to make conditions on the planet habitable for life. In this way, a sort of homeostasis would be sought by the self-organizing geophysical/physiological system of Earth.

In recent years, the Gaia hypothesis has found its place in Earth Systems Science as the realization that there is just one global ecosystem, containing the entirety of resources and all living organisms, all interacting with each other in multiple regulatory cycles. These ideas have been connected to the Darwinian theory of evolution via natural selection [28, 29], providing a mechanism by which such a stable state can be assumed to have emerged.

Other examples of natural self-organizing systems can be found in Table 1.

## V Examples of artificial self-organizing systems

There are numerous examples of man-made systems or systems which involve man that exhibit self-organization phenomena. Among them are traffic patterns, self-organizing neural networks, cellular phone networks or the development of web communities.

The example we shall briefly discuss is that of traffic flow patterns. Macroscopic patterns of traffic jams on highways have been observed and experimentally examined [30]. Their appearance is closely related to traffic density, the

model of behaviour for drivers and the traffic flow that this allows [31]. Traffic flow is an open system, and it develops waves of traffic jams (solitons) excited by the density of traffic. Transitions between different traffic flow patterns have been considered as phase transitions, typical products of self-organization in the non-living world.

A number of examples of self-organizing systems from different fields is given in Table 1, lower section.

## VI Explanatory concepts of self-organization

Despite half a century of inquiry, the theory of self-organizing systems is still in its infancy. There is no "standard model" of SOS, only various aspects emphasized by different researchers. Here we shall discuss the most important of these.

### VI.1 Non-equilibrium Thermodynamics

Thermodynamics has been concerned with the notion of order and disorder in physical systems for more than a century. The theory of self-organization has to address fundamental issues of this field. The most important question in this regard is, how order can arise through self-organization.

Classical thermodynamics has focussed on closed systems, i.e. systems isolated from external influence in the form of matter and energy flow. This allowed to understand the processes involved when a system evolves undisturbed. A key result of this inquiry is the second law of thermodynamics, originally formulated by Carnot and later refined by Clausius in the 19th century. It states that "any physical or chemical process under way in a system will always degrade the energy". Clausius introduced a quantitative measure of this irreversibility by defining entropy:

$$S \equiv \int dQ/T \quad (1)$$

with Q the heat energy at a given temperature T. In any process of a closed system, entropy always rises

$$\frac{dS}{dt} \geq 0. \quad (2)$$

According to Eddington, 1928 [32] this universal increase in entropy "draws the arrow of time" in nature.

Boltzmann had reformulated entropy earlier in terms of the energy microstates of matter. In his notion, entropy is a measure of the number of different combinations of microstates in order to form a specific macrostate.

$$S = k_B \ln(W) \quad (3)$$

| System                | Flow                           | Self-organizing entities                                          | Emergence                             |
|-----------------------|--------------------------------|-------------------------------------------------------------------|---------------------------------------|
| Atmosphere            | Solar energy                   | Gas molecules                                                     | Patterns of atmospheric circulation   |
| Climate               | Energy                         | Weather conditions<br>(humidity, precipitation, temperature, ...) | Distribution patterns                 |
| Liquid between plates | Heat                           | Particle circulation                                              | Movement patterns                     |
| Laser                 | Excitation energy              | Phase of light waves                                              | Phase-locked mode                     |
| Reaction vessel       | Chemicals for BZ reaction      | Chemical reactions                                                | Patterns of reaction fronts           |
| Neural networks       | Information                    | Synapses                                                          | Connectivity patterns                 |
| Living cells          | Nutrients                      | Metabolic reactions                                               | Metabolic pathways / network patterns |
| Food webs             | Organisms of different species | Species rank relation                                             | Networks of species                   |
| Highway traffic       | Vehicles                       | Distance of vehicles                                              | Density waves of traffic              |
| City                  | Goods, information             | Human housing density                                             | Settling patterns                     |
| Internet              | Computer nodes                 | Connections between nodes                                         | Network connection pattern            |
| Web                   | Information posted in websites | Links between websites                                            | Patterns of web communities           |

**Table 1.** Examples of Self-Organizing Systems.

with  $k_B$  Boltzmann's constant and  $W$  the thermodynamic probability of a macrostate. He argued that the macrostate with most microstates (with maximum entropy) would be most probable and would therefore develop in a closed system. This is the central tenet of equilibrium thermodynamics.

More interesting phenomena occur if the restrictions for isolation of a system are removed. Nicolis and Prigogine [11] have examined these systems of non-equilibrium thermodynamics which allow energy and matter to flow across their boundary. Under those conditions, total entropy can be split into two terms, one characterizing internal processes of the system,  $d_i S$  and one characterizing entropy flux across the border  $d_e S$ . In a generalization of the second law of thermodynamics, Prigogine and Nicolis postulated the validity of the second law for the internal processes,

$$\frac{d_i S}{dt} \geq 0. \quad (4)$$

but explicitly emphasized that nothing can be said about the sign of the entropy flux. Specifically, it could carry a negative sign and it could be larger in size than the internal entropy production. Since the total entropy is the sum of both parts, the sign of the total entropy change of an open system could be negative,

$$\frac{dS}{dt} = \frac{d_i S}{dt} + \frac{d_e S}{dt} < 0 \quad (5)$$

a situation impossible in equilibrium thermodynamics. Thus, increasing order of the system considered would be possible through export of entropy. Self-organization of a system, i.e. the increase of order, would not contradict the second law of thermodynamics. Specifically, the non-equilibrium status of the system could be considered a source of order.

Even in the distance from thermodynamic equilibrium, however, certain stable states will occur, the *stationary* states. These states assume the form of *dissipative structures* if the system is far enough from thermodynamic equilibrium and dominated by non-linear interactions. The preconditions for dissipative structures can be formulated as follows:

1. The system is open.
2. The inner dynamics is mainly non-linear.
3. There are cooperative microscopic processes.
4. A sufficient distance from equilibrium is assumed, e.g. through flows exceeding critical parameter values.
5. Appropriate fluctuations appear.

If those conditions are fulfilled, the classical thermodynamic branch of stationary solutions becomes unstable and dissipative structures become stable system solutions.

## VI.2 Synergetics

Prigogine's description of dissipative structures is formally limited to the neighborhood of equilibrium states. As Haken pointed out, this is a severe restriction



on its application and in particular precludes its formal application to living systems. Instead, Haken proposed *order parameters* and the *slaving principle* as key concepts for systems far from equilibrium. Let the time evolution of a continuous dynamical system is described by

$$\frac{d\mathbf{q}}{dt} = \mathbf{N}(\mathbf{q}, \alpha) + \mathbf{F}(t) \quad (6)$$

where  $\mathbf{q}(t) = [q_1(t), q_2(t), \dots, q_N(t)]$  is the system's state vector and  $\mathbf{N}$  is the deterministic part of the system's interaction whereas  $\mathbf{F}$  represent fluctuating forces, and  $\alpha$  are the so-called control parameters. Then the stable and unstable parts of the solution can be separated by linear stability analysis, as can the time dependent and time independent parts. As a result, the solution can be written as

$$\mathbf{q}(t) = \mathbf{q}_0 + \sum_u \xi_u(t) \mathbf{v}_u + \sum_s \xi_s(t) \mathbf{v}_s \quad (7)$$

$\mathbf{v}_u, \mathbf{v}_s$  are the unstable and stable modes, respectively, and  $\xi_u(t), \xi_s(t)$  are their amplitudes. These amplitudes obey the following equations

$$\frac{d\xi_u}{dt} = \lambda_u \xi_u + N_u(\xi_u, \xi_s) + \mathbf{F}_u(t) \quad (8)$$

$$\frac{d\xi_s}{dt} = \lambda_s \xi_s + N_s(\xi_u, \xi_s) + \mathbf{F}_s(t) \quad (9)$$

with  $\lambda_u, \lambda_s$  characterizing the linear part of the equations and function  $N$  summarizing the non-linear deterministic components. The slaving principle formulated by Haken now allows to eliminate the stable mode development by expressing them as a function of unstable modes

$$\xi_s(t) = f_s[\xi_u(t), t]. \quad (10)$$

Thus, the unstable modes (order parameters) enslave the stable modes and determine the development of the system's dynamics. This result is useful both to describe *phase transitions* and *pattern formation* in systems far from equilibrium.

Synergetic concepts have been applied in a variety of disciplines [33].

### VI.3 Chaos and Complexity

The treatment of *chaotic systems* has been derived from non-linear system theory. Chaotic systems are usually low-dimensional systems which are unpredictable, despite being deterministic. The phenomenon was originally discovered by the meteorologist E. Lorenz in 1963 [34], although H. Poincare in 1909 was aware of the possibility of certain systems to be sensitive to initial conditions [35]. The reason for the difficulty to predict their behavior stems from the fact that initially infinitesimal differences in trajectories can be amplified by non-linear interactions in the system. These instabilities, together with the lack of

methods for solving even one-dimensional non-linear equations analytically, produce the difficulties for predictions. Modern theory of deterministic chaos came into being with the publication of a seminal article by May in 1976 [36].

Complex systems, on the other hand, have many degrees of freedom, mostly interacting in complicated ways, i.e. they are high-dimensional. All the more astonishing is the fact that our world is not totally chaotic in the sense that nothing can be predicted with any degree of certainty. It became apparent, that chaotic behavior is but one of the ways non-linear dynamical systems behave, with other modes being complex attractors of a different kind.

*Complexity* itself can be measured, notably there exist a number of complexity measures in computer science, but describing or measuring complexity is not enough to understand complex systems.

#### VI.4 Self-organized Criticality

For particular high-dimensional systems, Bak et al. [43] have suggested a dynamic system approach toward the formation of fractal structures, which are found to be wide-spread both in natural and artificial environments. His canonical example was a pile of sand. They examined the size and frequency of avalanches under certain well-prepared conditions, notably that grains of sand would fall on the pile one by one. This is an open system with the forces of gravity and friction acting on the possibly small fluctuations that are caused by deviations in the hitting position of each grain of sand. He observed how the grains would increase the slope of the sand pile until more or less catastrophic avalanches developed.

Bak suggested the notion of *self-organized criticality* (SOC) as a key concept which states that large dissipative systems drive themselves to a critical state with a wide range of length and time scales. This idea provided a unifying framework for the large-scale behavior in systems with many degrees of freedom. It has been applied to a diverse set of phenomena, e.g. in economic dynamics and biological evolution. SOC serves as an explanation for many power-law distributions observed in natural, social and technical systems, like earthquakes, forest fires, evolutionary extinction events, and wars. As opposed to the widely studied low-dimensional chaotic systems, SOC systems have a large number of degrees of freedom, and still exhibit fractal structures as are found in the extended space-time systems in nature.

#### VI.5 The Hypercycle

In a series of contributions since 1971, Eigen and Schuster have discussed particular chemical reaction systems responsible for the origin, self-organization and evolution of life [37–40]. By considering *autocatalytic sets* of reactions they arrived at the most simple form of organisation, the *hypercycle*, which is able to explain certain aspects of the origin of life. They have considered a chemical reaction system composed of a variety of self-reproductive macromolecules and

energy-rich monomers required to synthesize those macromolecules. The system is open and maintained in a non-equilibrium state by a continuous flux of energy-rich monomers. Under further assumptions they succeeded in deriving Darwinian selection processes at the molecular level. Eigen and Schuster have proposed rate equations to describe the system.

The simplest system realizing the above mentioned conditions can be described by the following rate equations

$$\frac{dx_i}{dt} = (A_i Q_i - D_i)x_i + \sum_{k \neq i} w_{ik} x_k + \Phi_i(\mathbf{x}) \quad (11)$$

where  $i$  enumerates the individual self-reproducing units and  $x_i$  measures their respective concentrations. Metabolism is quantified by the formation and decomposition terms  $A_i Q_i x_i$  and  $D_i x_i$ . The ability of the self-reproducing entities to mutate into each other is summarized by the quality factor for reproduction,  $Q_i$ , and the term  $w_{ik} x_k$  which takes into account all catalytic productions of one sort using the other.  $A_i, D_i$  are rate constants for selfreproduction and decay respectively. The flow term  $\Phi_i$  finally balances the production / destruction in this open system in order to achieve  $\sum_k x_k = const.$

By introducing a new feature called excess production

$$E_i \equiv A_i - D_i \quad (12)$$

and its weighted average

$$\bar{E}(t) = \sum_k E_k x_k / \sum_k x_k \quad (13)$$

and symbolizing the "intrinsic selection value" of a sort  $i$  by

$$W_{ii} = A_i Q_i - D_i \quad (14)$$

one arrives at reduced rate equations

$$\frac{dx_i}{dt} = (W_{ii} - \bar{E})x_i + \sum_{k \neq i} w_{ik} x_k. \quad (15)$$

These equations can be solved under certain simplifying assumptions and notably yield the concept of a *quasi-species* and the following extremum principle: A quasi-species  $y_i$  is a transformed self-replicating entity with the feature that it can be considered as a cloud of sorts  $x_i$  whose average or consensus sequence it is. The extremum principle reads: Darwinian selection in the system of quasi-species will favor that quasi-species which possesses the largest eigen-value of the rate equation system above.

## VI.6 The Origin of Order

In the 1990s Kauffman [41] pointed out one of the weaknesses of Darwinian theory of evolution by natural selection: It cannot explain the 'origin of species'

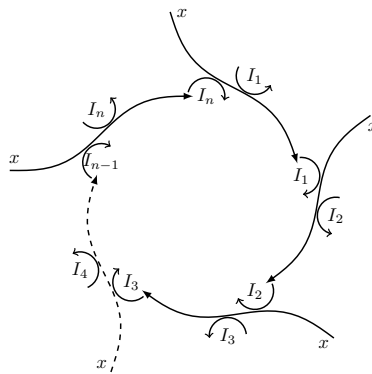


Figure 1: a

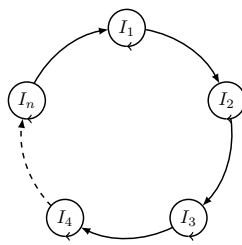


Figure 2: b

**Fig. 1.** The Hypercycle. Reproduced from Eigen & Schuster [15].

but rather only their subsequent development. Kauffman instead emphasized the tendency of nature to constrain developments along certain paths, due to restrictions in the type of interaction and the constraints of limited resources available to evolution. In particular he held up the view that processes of spontaneous order formation conspire with the Darwinian selection process to create the diversity and richness of life on Earth.

Previously, Kauffman had formulated and extensively studied [42] the NK fitness landscapes formed by random networks of  $N$  Boolean logic elements with  $K$  inputs each. Kauffman observed the existence of cyclic attractor states whose emergence depended on the relation between  $N$  and  $K$ , and the absolute value of  $K$ . In the case of large  $K$  ( $K \approx N$ ), the landscape is very rugged and behavior of the network appears stochastic. The state sequence is sensitive to minimal disturbances and to slight changes of the network. The attractor length is very large,  $\approx N/2$ , and there are many attractors. In the other extremal case,  $K = 2$ , the network is not very sensitive to disturbances. Changes of the network do not have strong and important consequences for the behavior of the system.

Kauffman proposed NK networks as a model of regulatory systems of living cells. He further developed the notion of a *canalizing function* that is a Boolean function in which at least one variable in at least one state can completely determine the output of the function. He proposed that canalizing functions are an essential part of regulatory genetic networks.

## VI.7 Emergence and Top-down Causation

The notion of *emergence* has been introduced in complex systems theory in order to explain the appearance of new qualitative features on the level of an entire system that could not be observed at the level of its components. Emergent behavior can be connected to the afore-mentioned complex attractors. It requires switching the level of description of behavior of a system, from local (component-centered) to global (system-centered), or at least to a meso-level (sub-system-centered). Emergent behavior happens when

- a) the system shows qualitatively new behavior on a higher level of description which
- b) could not have been easily predicted from the interactions of components at the lower level (obeys a non-linear relationship)
- c) is the result of a self-organization process.

Emergence is strongly related to self-organization. It is often understood as a pattern formation process. While it essentially has to do with changing the perspective and looking at the system at a different level, it concerns itself with a change in behavior (e.g. the system is getting more organized, shows new coordinated modes of behavior). It has been further conjectured that there is top-down causation, i.e. the structures forming on the higher level of the system are able to affect the lower levels (system components) and influence them in a way that stabilizes the newly emergent behavior. Haken could show in the context of Synergetics that this phenomenon exists. *Top-down causation* is believed to

be an important source of complexity, especially in living systems, because it stabilizes patterns.

Self-organization draws heavily from this source of qualitative innovation in complex systems.

## VII Modeling Methods

A formal model is a simplified mathematical or algorithmic representation of a system. Often it has been simplified to the point of a caricature, and this has to be born in mind when making conclusions about the consequences of model predictions. No model can predict beyond the limits of its approximations.

### VII.1 Mean-field Methods

One of the most important methods used to model complex systems is tied to the notion of dynamical systems. Dynamical systems are systems whose time development is accessible to a description by state changes. It entails the existence of a state space in which these changes can be traced and quantified.

Mean-field methods of description focus on average behavior. They abstract away from the local correlations between a system's elements and describe only long-range changes. For instance, the behavior of a planet could be described as a point on its trajectory around the star it circles. Detailed interactions of its atmosphere would not be part of that description.

Mean-field methods are formulated in the form of time-dependent differential or difference equations which can be solved under certain conditions and predict the behavior of a system in its state space.

Assuming that the state of a system can be subsumed in a vector of state variables  $\mathbf{x}$  whose values are observable and depend on time, we can generally formulate an equation for continuous time development as:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}) \quad (16)$$

If time does not develop continuously, a discrete (iterative) equation can be used to describe system behavior:

$$\mathbf{x}_{t+1} = \mathbf{f}(\mathbf{x}_t) \quad (17)$$

The notion is that the state of the system at time  $t$ ,  $\mathbf{x}(t)$  or  $\mathbf{x}_t$ , is everything that is of interest and can be known about the system. It turned out that with this extreme simplification many systems became treatable that would have otherwise resisted quantitative treatment. The non-linear nature of many of the state development equations, however, and the high-dimensionality of state space vectors often constitute prohibitive hurdles to exact or even approximate mathematical solution of these equations.

As a result, algorithmic approaches for modeling self-organizing systems have become more prevalent in recent years.

## VII.2 Agent-based Models

A very general class of algorithmic systems is subsumed under the term agent-based models. In these systems, individual entities are modeled that interact with each other. Thus, the approximation of average behavior, and the interest for long-term behavior only is abandoned in favor of a microscopic description of the elements of a system and their interactions. The abstraction of features of a system is achieved through the assumption of rules of behavior of the agents, including their interaction behavior. Agent-based systems must be implemented as computational systems, and run on a computer to obtain results. Agents are assigned states, and transition rules between states, depending on interacting agents, and then these rules are executed in parallel over the set of agents under consideration.

**Cellular Automata** A particular subset of agent-based models is the class of cellular automata introduced by von Neumann [47] going back to lattice networks of Ulam. The agents of cellular automata are placed on a grid of cells and allowed to assume a finite number of states. Interactions are determined by state transition rules and the definition of a neighborhood, which determines the interactivity of the cellular automaton. Many variants of cellular automata exist, differing in the number of dimensions of the grid, the number of states, the sort and distribution of transition rules and the nature of the neighborhood.

A typical cellular automata model might, e.g. consist of digital cells (allowing only two states, "ON" and "OFF"), homogeneous and deterministic transition rules between states, a one-dimensional grid, and nearest-neighbor interactions. Cellular automata of this type have been thoroughly examined in [44, 45] and show a surprising variety and richness in behavior.

In a cellular automaton like LIFE, for instance, one can observe how macroscopic and mesoscopic structures appear through self-organization, that is, as a process determined solely by the local interaction of the CA's elements. Some structures, e.g. spiral waves, are more resilient against perturbation than others, e.g. glider canons. A moving structure like a glider can be interpreted as an emergent phenomenon as it does not seem to be present on the microscopic scale (single CA cells do not move).

**Graphs and Networks** A more general class of automata can be formulated if the notion of cellular neighborhood is abandoned. Instead of a rigidly defined grid, a graph or network of automata connected through edges to other automata is introduced. Each node of the graph / network represents an automaton, with interactions allowed via edges.

The notion of a graph is, however, more general, and allows other agent-based systems to be simulated. For instance, the nodes of a graph might represent species of an ecosystem interacting with other species (connected by edges). Each species might be represented by a state counting the number of individuals of that species. Nodes might further hold information on particular features of

individuals, and possibly their variants. Explicit simulations of such systems have been considered in the context of "Artificial Chemistries" [46].

In recent years, the structure and dynamics of networks has been a major focus of interest in the scientific community. Network science has become a converging point for different disciplines interested in modelling complex behavior.

### VII.3 Observables

Self-organizing phenomena rest on the appearance of particular sets of behaviors. If ever they are to be understood, a clear notion of observable quantities needs to be developed that allows a proper description of the behavior of such systems. At present, no such canonical set of observables exists, owing to the bewildering variety of systems that show signs of self-organization. However, one can discern a number of different measures and observables that might form the core of such a set [49].

**Entropic and Information Theoretic Measures** One class of observables can be considered entropic and information-theoretic measures. These measures have in common a statistical root, and seek to describe a self-organizing system in terms of the order (or disorder) that develops over time [48].

**Stability Measures** Another class of observables can be discussed as stability measures. In this class, systems are sought to be disturbed from their regular behavior in order to obtain a clearer idea of their resilience.

**Scaling Measures** A further class of observables can be attached to features of scaling. Both theoretical and experimental approaches can be used to vary the number of dimensions, number of equations/agents, number and complexity of interactions, etc, in these systems. Scaling behavior can then be observed for particular quantities and systems classified accordingly.

**Patterns and Flows** The defining observables of a self-organizing system are patterns. These refer to the collective behavior of the elements of a system, differentiating them from noise. If individual entities would not show such correlations in their behavior, self-organization could not be observed. Patterns can be described in a variety of ways, e.g. as multidimensional vectors, using spatial and temporal coordinates. If patterns change dynamically one can speak of flows.

The central tenet of self-organization is that systems exist whose pattern forming tendencies are determined by themselves, and not by an outside agency.

## VIII The Role of Self-organization in Science, the Social Sciences and Engineering

Self-organization as a concept has assumed center stage in Science. With the advent of nonlinear systems and studies on complex systems in non-equilibrium sit-



uations, the explanatory power of self-organization now permeates every branch of scientific enquiry.

From structure formation at the level of super-galactic clusters, even starting from the development of the entire universe, down to microscopic particles and their interaction patterns, self-organizing phenomena have been postulated, theorized, observed and confirmed.

In particular the origin and evolution of life have been studied under the aspect of self-organization. Within Biology, the developmental process of organisms as well as their metabolisms, growth and learning have been identified as self-organizing processes.

In the humanities, the idea of self-organization has taken roots, although the paradigm is far from being fully recognized yet. Since the 1990s the origin and development of languages has been an object of study under the premise of self-organization. In social science the concept of self-organization has been studied since a number of years, due to the obvious fact that interaction between social actors generate a society. Even in psychology, self-organizing principles begin to appear.

Economy and Management Science have taken notice of the concept, and a growing number of enterprise concepts promote the idea of a firm as a self-organizing entity.

Finally, Philosophy has embraced the concept of self-organization and connected it to earlier thoughts on the development of the scientific process and epistemology. Whitehead put forward his process philosophy, and Smuts, already in the 1920s, promoted the notion of holism which has strong connections to self-organization. Evolutionary epistemology was formulated as a response to traditional epistemology and emphasizes the aspect of natural selection affecting senses and cognitive abilities.

Engineering is beginning to grasp the ubiquity of self-organization in Nature. Specifically in the area of nanotechnology the concept is used extensively for the purpose of self-assembly of molecular entities. At nanoscales, it is very difficult to directly specify the structuring behavior of entities. As a result, self-organizing properties of matter are used to the advantage of the structural outcome.

Different kinds of infrastructure networks have been recognized as self-organizing, and Engineering begins to make use of the tendency of networked systems to self-organize.

In the area of adaptation, there exists a long tradition of making use of self-organization principles. The self-organizing feature map, introduced by Kohonen, has been a key step forward in the domain of unsupervised learning of artificial neural networks.

## **IX Open Issues and Future Directions**

So far, there is no unique theory of self-organization. Over the course of many years different approaches have been used, but a coherent picture has not yet emerged.

An important open question in the area of the mathematical basis for self-organization is the formulation of a theory of *constructive (evolutionary) systems*, that is systems which, in the course of their development, generate new elements that subsequently interact with elements already created earlier.

Another question aims at the *raison d'être* of *hierarchical systems*. Why do they form, how do they structure themselves, and what would be possible to apply from these principles in Engineering? Notably, how would one build self-organizing systems such that they do something useful? How could they be controlled?

In Science, the build-up of complexity remains a controversial issue. Is it true that evolution of the universe tends to increase complexity, or is there no tendency of complexity increase at all? What are the mechanisms by which Nature increases complexity, if any? How could we apply this knowledge in planning and managing complexity in the human world?

A wealth of questions remains, and it is anticipated that the 21st century will shed light on at least a few of them.

## References

1. L. von Bertalanffy, *General System Theory: Essays on its Foundation and Development*, George Braziller, New York, 1968
2. N. Wiener, *Cybernetics:: Or Control and Communication in the Animal and the Machine*, MIT Press, Cambridge, MA, 1965
3. W.R. Ashby, *Principles of the Self-Organizing Dynamic System*. J. Gen. Psychol., **37** (1947) 125-128
4. B.G. Farley, W.A. Clark, *Simulation of self-organizing systems by digital computer*. IRE Transactions on Information Theory, **4** (1954) 76-84.
5. W.R. Ashby, *Design for a Brain: The Origin of Adaptive Behavior*. Chapman and Hall, London, 1960.
6. H. von Foerster, *On Self-organizing systems and their environments*. In: *Self-Organizing Systems*, M.C. Yovits and S. Cameron (Eds.), Pergamon Press, NY, 1960, 31-50.
7. H. Haken, *Synergetics - An Introduction*, Springer, Berlin, 3rd ed, 1983
8. S. Kauffman, *The Origins of Order*, Oxford University Press, Oxford, 1993
9. M. Conrad *Adaptability*, Plenum Press, New York, 1983
10. T. Kohonen, *Self-Organizing Maps*, Springer Series in Information Sciences, 30 Springer, Berlin, 2000
11. G. Nicolis, I. Prigogine, *Self-Organization in Nonequilibrium Systems* Wiley, New York, 1977
12. H. Haken, *Information and Self-Organization : A Macroscopic Approach to Complex Systems*, Springer Series in Synergetics, Springer, Berlin, 2nd ed, 2000
13. R. Thom, *Structural Stability and Morphogenesis: An Outline of a General Theory of Models*, Addison-Wesley, Reading, MA, 1989
14. B. Mandelbrot, *The fractal Geometry of Nature*, W.H. Freeman, San Francisco, 1982
15. M. Eigen, P. Schuster, *The Hypercycle*, Springer, Berlin, 1979
16. R. Rosen, *Life Itself: A Comprehensive Inquiry into the Nature, Origin, and Fabrication of Life*, Columbia University Press, New York, 1991

17. C. von der Malsburg, *Self-organization of orientation sensitive cells in the striate cortex*. *Kybernetik* **14** (1973) 85-100
18. R. Linsker, *Self-organization in a perceptual network*. *Computer* **21** (1988) 105-117
19. H. von Foerster, *Understanding Understanding*, Springer, New York, 2002
20. H. Maturana, F. Varela, *Autopoiesis and Cognition*, Reidel, Dordrecht, 1979
21. S. Camazine, J. Deneubourg, N. Franks, J. Sneyd, E. Bonabeau, G. Theraulaz, *Self-Organization in Biological Systems*. Princeton University Press, Princeton, 2000
22. R. Sole, J. Bascompte, *Self-Organization in Complex Ecosystems*, Princeton University Press, 2006
23. N. Luhmann, *Social Systems*, Stanford University Press, 1995
24. S. Focardi, S. Cincotti, M. Marchesi, *Self-organization and market crashes*, *J. of Economic Behavior and Organization*, **49** (2002) 241-267
25. J. Portugali *Self-Organization and the City*, Springer, Berlin, 2000
26. S. Bruckner, G. di Marzo Serugendo, A. Karageorgos, R. Nagpal (Eds.) *Engineering Self-Organizing Systems*, Springer, Heidelberg, 2005
27. J.E. Lovelock, L. Margulis, *Atmospheric homeostasis by and for the biosphere: The Gaia hypothesis*, *Tellus* **26** (1974) 2-9
28. T.M. Lenton, *Gaia and natural selection*, *Nature* **394** (1998) 439-447
29. M. Staley, *Darwinian Selection Leads to Gaia*, *J. Theor. Biol.* **218** (2002) 35-46
30. BS Kerner, *Experimental Features of Self-Organization in Traffic Flow*. *Physical Review Letters* **81** (1998) 3797-3800
31. M. Treiber, D. Helbing, *Explanation of Observed Features of Self-Organization in Traffic Flow*. Arxiv preprint cond-mat/9901239 (1999)
32. A.S. Eddington *The Nature of the Physical World* Gifford Lectures, Cambridge University Press, 1928
33. H. Haken, *Synergetics. Introduction and Advanced Topics.*, Springer, Berlin, 2004
34. E. Lorenz, *Deterministic Nonperiodic Flow*, *J. of Atmospheric Science* **20** (1963) 130-141
35. H. Poincare, *Science et Methode*, Flammarion, Paris, 1909.
36. R.M. May, *Simple Mathematical Models with very complicated Dynamics*, *Nature* **261** (1976) 459-467.
37. M. Eigen, *Selforganization of Matter and the Evolution of Biological Macromolecules*, *Naturwissenschaften* **58** (1971) 465-523.
38. M. Eigen, P. Schuster, *The hypercycle. A principle of natural self-organization. Part A: Emergence of the hypercycle*. *Naturwissenschaften* **64** (1977) 541-565
39. M. Eigen, P. Schuster, *The hypercycle: a principle of natural self-organization, part B*. *Naturwissenschaften* **65** 7-41
40. M. Eigen, P. Schuster, *The hypercycle: a principle of natural self-organization, part C* *Naturwissenschaften* **65** 341-369
41. S.A. Kauffman, *The Origins of Order*, Oxford University Press, 1993
42. S.A. Kauffman, *Metabolic Stability and epigenesis in randomly constructed nets*. *J Theor. Biol.* **22** (1969) 437-467.
43. P. Bak, C. Tang, K Wiesenfeld, *Self-organized criticality*. *Physical Review* **A38** (1988) 364-374
44. S. Wolfram, *Statistical Physics of Cellular Automata*. *Reviews of Modern Physics* **55** (1983) 601-644
45. S. Wolfram, *Cellular Automata as Models of Complexity*. *Nature* **311** (1984) 419-424
46. P. Dittrich, J. Ziegler and W. Banzhaf, *Artificial Chemistries - A Review*. *Artificial Life* **7** (2001) 225-275

47. J. von Neumann, *Theory of Self-Reproducing Automata*. Univ. of Illinois Press, Chicago, 1966.
48. D. Polani, *Measuring Self-Organization via Observers*. In: W. Banzhaf et al. (Eds.), *Advances in Artificial Life*, Springer LNAI **2801** (2003) 667-675.
49. C.R. Shalizi, K.L. Shalizi and R. Haslinger, *Quantifying Self-Organization with Optimal Predictors*. Phys.Rev.Lett. **93** (2004) 118701.

## Further Reading

1. P. Bak, *How Nature Works - The Science of Self-Organized Criticality*, Springer, New York, 1996
2. Y. Bar-Yam, *Dynamics of Complex Systems*, Westview Press / Perseus Books, New York, 2003
3. N. Boccarda, *Modelling Complex Systems*, Springer, New York, 2004
4. G. Edelman, *Neural Darwinism*, Basic Books, New York, 1987
5. C. Hemelrijk (Ed.), *Self-organization and Evolution of Social Systems*, Cambridge University Press, 2005
6. J. Holland, *Emergence: From Chaos to Order*, Oxford University Press, 2000
7. E. Jantsch, *The Self Organizing Universe : Scientific and Human Implications*, Pergamon Press, New York, 1980
8. S. Johnson, *Emergence*, Scribner, New York, 2001
9. S. Kelso, *Dynamic Patterns: The Self-organization of Brain and Behavior*, MIT Press, Cambridge, 1995
10. J.E. Lovelock, *The Ages of Gaia*, W.W. Norton, 1995
11. J. Mingers, *Self-Producing Systems*, Plenum Press, New York, 1995
12. H. Morowitz, *The Emergence of Everything*, Oxford University Press, 2002
13. R. Rosen, *Essays on Life Itself*, Columbia University Press, 1999
14. D. Sornette, *Critical Phenomena in Natural Sciences*, Springer, Berlin, 2003

## See also

- Open Systems
- System Modelling
- Non-equilibrium systems
- Entropy
- Cellular Automata
- Swarm Intelligence
- Biological Development and Evolution
- Social Systems
- Information Systems
- Communication Systems