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ORIGINAL ARTICLE

Coevolutionary opinion dynamics with sparse interactions in open-ended societies

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Abstract

Opinion dynamics is a crucial topic in complex social systems. However, existing models rarely study limited information accessibility, sparse interactions, and the coevolution of opinion and an open-ended structure. In this paper, we propose the **S**parse **CO**evolutionary **O**pen-**E**nded (**SCOOE**) model. We address the sparse interaction limitation through extrinsic collective interaction and intrinsic observation based on incomplete neighborhood information. We also consider the coevolution of opinion and open-ended structure by studying structure-opinion co-dynamics when dissidents are leaving and when newcomers with novel opinions are joining. From an opinion dynamics perspective, we find that the proposed mechanisms effectively form lean and fast decision strategies to reduce conflicts under uncertainty. The model is robust in boosting and enhancing a global consensus with only small odds of extreme results. The structure evolves toward a small-world network. We find that an emergent dialectic relationship exists between community segregation and community cohesion viewed from a structural dynamics perspective. We also study the influence of agent heterogeneity under different cognitive ability distributions.

Keywords Opinion dynamics · Sparse interactions · Collective decision-making · Open-endedness · Coevolution

Introduction

The study of opinion dynamics, i.e., the study of the formation and dynamics of public opinions, is a crucial research topic in complex systems and social networks. The topic has been widely explored for several decades in theoretical models and real-world applications among different disciplines, including social science, control engineering, statistical physics, and computer science. Elucidation of the mechanisms behind macro-level opinion dynamics is vital for understanding social interactions/dynamics, complexity, distributed control, and decision-making. It also holds valuable lessons to apply to real-world empirical studies and applications like marketing and social media [37].

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An agent-based model is a type of computational model focusing on the bottom-level (micro) interactions and their effects on the holistic (macro) system. It is a powerful tool to study evolutionary phenomena in both the natural and social sciences. Many classic agent-based models have been explored under various assumptions to study opinion dynamics from different perspectives. For example, the Hegselmann–Krause model [23] studies opinion polarization with the bounded-confidence assumption, i.e., agents interact only if their opinions are sufficiently close to each other by falling within a confidence interval. The Sznajd model and its variations [51] study the evolution of consensus in a closed society through majority voting. In that model, a focal agent polls its complete neighborhood (i.e., the group of agents sharing connections with the focal agent in the social network) and selects the opinion of the majority. However, some assumptions in existing models, e.g., polling the complete neighborhood, seem to be no longer suitable, notably when people with bounded rationality only have a partial view and cannot access the complete neighborhood information in their social networks. Meanwhile, when we interact with neighbors, the literature from psychology suggests that we are mainly concerned with the overall opinion of neighbors (e.g., a joint opinion through collective decision-making), and we adjust our own opinions according to this feedback [15]. Some other work uses the bounded-confidence assumption [52] applying dense interactions and serial opinion updates through interacting with all selected neighbors. While existing models thoroughly describe the dynamics of opinions and interactions, they ignore the built-in structural dynamics caused by opinion dynamics and open-endedness, e.g., through the effects of newcomers, leavers, and their impact on structure-opinion coevolution.

To address the preceding limitations, this paper's contribution can be summarized as follows.

– We develop the **S**parse **CO**evolutionary **O**pen-**E**nded (**SCOOE**) model, and then use this model to answer two questions: How do opinions evolve under conditions of sparse interactions and incomplete information, and how do opinion dynamics guide the coevolution of an open-ended society?
– We propose a framework for collective sparse interaction among bounded-rational agents. The focal agent in our model has a limited view of a partial neighborhood. It is first assimilated into the extrinsic environment via collective information derived from its incomplete neighborhood rather than interacting with all neighbors or polling the entire neighborhood. Then, it is driven by intrinsic motivation to voluntarily adjust its opinion by observing the incomplete neighborhood and social learning without direct interactions.
– We examine structural co-dynamics and pay close attention to the open-endedness property. Opinions and the open-ended society coevolve in the model: The opinion dynamics affect which agent will become a dissident, exiting society, and the associated structural dynamics. Meanwhile, a joiner with possibly novel opinions affects its neighbors, and its opinion might cascade through the system via sparse interactions. We also carry out experiments to study the structural dynamics and opinion dynamics searching for answers to the two questions.

It is also worth noting that a large body of computer science research (e.g., multi-agent system community) aims to raise the cooperation rate or speed to state of the art (SOTA) through advanced deep learning methods [24]. We are investigating a relatively unexplored social phenomenon, namely, the sparse interactions and coevolution of structure and opinion under open-ended networked assumptions. As discussed in our contribution, we design a new model to improve the limitation of prior works, and it yields theoretically and empirically plausible results. However, we do not attempt to demonstrate the superiority of this model over competing models.

The rest of the paper is organized as follows: We critically review the related literature and comment on the current limitations in "Previous work". Our SCOOE model is then introduced in "Sparse coevolutionary open-ended (SCOOE) model", including agent design/model initialization ("Agent design and model initialization") and model dynamics ("Model dynamics"). Experimental results are reported in Sect. "Experiments". In "Discussion", we present a comprehensive discussion of the model. We conclude by pointing out future research directions in "Conclusions".

Previous work

Research in opinion dynamics models is mainly conducted in two directions. If the focal entity (agent) chooses opinions from a discrete set of opinions, the model is called a discrete-opinion model [13], whereas if the focal agent's opinions are represented by a continuous interval, the model is called a continuous-opinion model [12,23]. In the general framework of an opinion dynamics model, agents update their opinions according to interactions with randomly chosen others or with connected neighbors in a networked structure. Stable patterns of opinions will evolve over time, e.g., group agreement, polarization, or multiple local opinions distributed in different communities [34].

Recent work has revised the following three perspectives to improve the two basic models: (i) the representation of opinions, (ii) the opinion fusion methods, and (iii) the heterogeneity of agents. Some work has explored novel representations of opinions with multiple topics [55]. Liu et al. [36] propose a fuzzy set-based representation of opinions reflecting the attitudes of tolerance and stubbornness of humans. Some work has introduced different mechanisms for opinion fusion, e.g., the multi-level bounded-confidence model [30], where society is divided into multiple subgroups with different levels of confidence intervals. There, agents prefer to only interact with others whose opinions are falling into their corresponding confidence intervals. Some work has introduced different types of heterogeneity, including antagonistic agents [32], leader–follower relationships [14], and stubborn agents [58].

Although this previous work has offered important insights, it also involves two important limitations. First, a widely adopted assumption in existing models is complete information, i.e., rational agents have perfect information to poll their neighborhoods or conduct interaction preferences (e.g., selecting neighbors) to update their opinions. For example, in the model of [48], the interaction partner is selected by a probability function generated from the opinion difference. However, this assumption is unrealistic, notably when the complete information of the neighborhood is inaccessible. We can indeed be affected by many people. However, the literature from psychology suggests that agents are primarily influenced by taking the collective opinions of others (e.g., neighbors in the social network) into account, rather than updating their opinions serially and densely by interacting with all their neighbors or certain selected neighbors with similar opinions [15,42]. Sparse interactions with incomplete information have rarely been studied previously [8,21].

Second, the majority of previous extensions are carried out in closed or static systems. Insufficient attention is paid to the built-in system dynamics. The majority of work on structural dynamics is concerned with manipulating structures. That is, how to influence the networked society toward consensus and, further, how to influence the agents toward a predetermined opinion via structural factors, such as adding a minimum number of edges to the network [7,22]. Rather than manipulating the structure to achieve more effective propaganda, we argue that it is worthwhile to consider an open-ended evolutionary system where new agents can join and dissidents can leave, and where agent opinions are updated continuously (opinion evolution) and the structure coevolves simultaneously with opinion evolution (the coevolution of structure and opinion). Coevolutionary phenomena have been extensively studied in the field of social networks, for example, the SIENA and ERGM models [49], but are rarely considered in models of opinion dynamics.

We address these limitations in the proposed SCOOE model, which takes into account the bounded rationality (e.g., incomplete information) of agents and the coevolution of structure and opinion in an open-ended society.

Sparse coevolutionary open-ended (SCOOE) model

This section describes the two main aspects of the proposed model, model initialization and model dynamics.

Agent design and model initialization

First, we describe the basic building blocks of the SCOOE model, heterogeneous agents and opinion representation.

Heterogeneous agents

The majority of previous work has aimed to assign agents' different role-based or function-based heterogeneity (e.g., leader–follower) to empower the model describing different real-world behaviors. In this paper, we ask the following question: What factors primarily influence the integration of a new opinion with an original opinion? Literature from psychology suggests that the most critical factor in opinion fusion is the built-in cognitive heterogeneity of humans. For instance, the Big Five personality traits model asserts that people's personality exists on a spectrum with multiple dimensions, e.g., inventive/curious v.s. consistent/cautious [27]. Cognitive dissonance theory describes the psychological stress when people are exposed to contradictory opinions and reconcile them to be consistent [34]. For agent i being exposed to a new opinion, we assume that agent i has a built-in probability of sticking to its opinion, i.e., a stubbornness probability w_i. It describes the degree to which an agent relies on its original opinion. In contrast, the complement of stubbornness, i.e., an openness probability $1 - w_i$, quantifies the degree to which agent i is willing to adopt a new opinion derived from the interaction with other agents. Heterogeneity is produced when agents hold different built-in cognitive features represented by different stubbornness (or openness) probabilities. We assume that stubbornness w_i follows a probability distribution in the population, like a Poisson or Gaussian distribution. In the experimental section, we report on the influence of different stubbornness distributions.

Opinions of agents

The opinion O_i of an agent i is represented by a real number in the continuous interval [0,1]. It describes the degree to which an agent believes the propagated information, e.g., news or rumors. A higher value of opinion O_i means that agent i believes the propagated information more strongly. Initially, each agent is assigned a random opinion, i.e., a random number in the range [0,1].

Note that the opinion represented by O_i is independent of the stubbornness probability. The former refers to the attitude of an agent toward news items. It will be expressed and updated by taking the opinions of others into account. The latter describes the personality and cognitive ability of an agent. It is built into an agent, only refers to the agent itself, and will not be expressed or changed. In the real world, people (e.g., agents i) could initially believe a piece of news by selectively collecting information themselves (confirmation bias, i.e., people collect information trying to follow their original beliefs or biases, e.g., original opinions). However, they might easily be persuaded to change their opinions through some new evidence or short-term interactions with others. In such a case, the initial opinion of an agent i is $O_i \approx 1$, while its stubbornness is $w_i \approx 0$. In contrast, agent i might disagree with the news and could be difficult to convince. In such a case, $O_i \approx 0$, while $w_i \approx 1$. It is common to find that stubbornness and belief-based opinions are not clearly differentiated in the existing work.

Model dynamics

Here, we describe the mechanisms of the SCOOE model dynamics, i.e., the sparse interaction protocol of opinion dynamics and the coevolution of opinion and open-ended

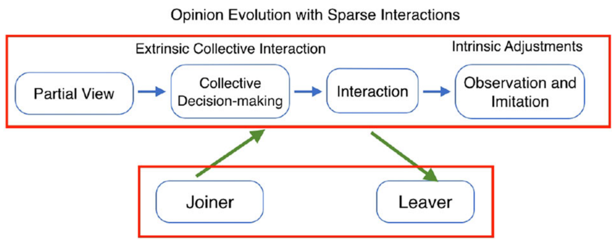


Fig. 1 The whole picture of the SCOOE model dynamics. Opinion dynamics with two components of sparse interactions (extrinsic and intrinsic forms): The focal agent with a limited view can only access a partial neighborhood. It aggregates a joint opinion of the incomplete neighborhood by collective decision-making. Then, the focal agent only takes this joint opinion from the extrinsic incomplete neighborhood into account by the interaction with the joint opinion, rather than by dense interactions with all neighbors or certain neighbors with similar opinions selected by polling the neighborhood. Imitation is also introduced to drive opinion intrinsic adjustments based on observation of the incomplete neighborhood environment without direct interactions with neighbors. The coevolution of open-ended structure and opinion: The opinion dynamics affect the leaver exiting society and associated structural dynamics. A joiner with a random opinion joins. It changes the structural features and the neighborhood settings, and the neighborhood settings in turn affect the opinion dynamics

structure. An overall picture of the SCOOE model is shown in Fig. 1.

Opinion dynamics with sparse interactions

We first discuss the opinions dynamics. The critical theme of opinion dynamics is sparse interaction and incomplete information. As we pointed out in the introduction and related work section, people do not serially poll the neighborhood in their social networks to update the opinion, but are mainly influenced through considering the joint opinion of others as their feedback [15]. This contrasts with most agent-based models which are formulated with such complete information assumptions, e.g., polling the entire neighborhood to select neighbors and conduct dense interactions serially to update opinions [15,42].

The literature from psychology suggests two types of motivations for humans to change behavior, extrinsic motivation (people are assimilated into extrinsic environments) and intrinsic motivation (people are motivated by internal desire) [45]. We take inspiration from this and assume two types of actions forming the sparse interaction, extrinsic collective interactions and an intrinsic observation mechanism. The interplay between these two actions enhances group opinion evolution. However, they play different roles in various stages of the model dynamics reported in the experimental section.

Extrinsic collective interaction with incomplete information

Here, we introduce a collective decision-making approach to incorporate the sparse joint opinion formation and interaction based on a limited neighborhood (i.e., incomplete information). Though several collective decision-making approaches have been proposed in discrete-opinion models, e.g., majority voting [9], this approach in continuous-opinion models has not been fully developed so far.

Suppose a focal agent i with a connection degree d_i in its social network is able to only access a random subset of neighbors i_1, i_2,..., i_j, where j is randomly chosen and satisfies $1 \le j \le d_i$. This assumption means incomplete information by a limited view and only partial access to neighbors, and it allows more dynamic interactions, e.g., an agent will not interact with its entire neighborhood. Agent i generates a joint opinion O_i^{Joint} of its random partial neighborhood, rather than by interacting with all its neighbors or certain neighbors with similar opinions serially. An intuitive way to generate a joint opinion is by taking the weighted average of the selected neighbors' opinions [16]. The weights assigned to different neighbors are proportional to their relative connection degree strength, as shown in Eq. 1, where d_{i_k} is the degree of neighbor i_k, $k \in [1, j]$

$$O_i^{Joint} = \sum_{k=1}^{j} \left(d_{i_k} / \sum_{k=1}^{j} d_{i_k} \right) \times O_{i_k}. \tag{1}$$

Thus, the more a neighbor is connected in the local network (measured by its relative connection strength), the greater its weight and influence on the joint opinion in the collective decision-making process.

Another critical factor in designing an interaction protocol is confirmation bias. That is, people collect and interpret information selectively by trying to follow their original bias (e.g., their original opinions) [44]. The most widely adopted interaction protocol with confirmation bias is a bounded confidence model where rational agents owning the perfect information poll their entire neighborhoods and select others to interact only if their opinions fall within a confidence interval [12,20,52]. Here, we take inspiration from game theory and model this as an opinion interaction game with confirmation bias among bounded-rational agents with limited information. Therefore, after generating a weighted joint opinion based on limited neighborhood information, agent i with opinion O_i receives a payoff R_i represented by Eq. 2

$$R_i = 1 - |O_i - O_i^{Joint}|. \tag{2}$$

Equation 2 means that if the opinion O_i of agent i is very different from the joint opinion in its selected neighborhood (the local environment), it receives a low payoff. Neighborhoods with more similar opinions are considered more trustworthy, thus, resulting in a higher payoff.

After considering the collective interaction by the game-playing and interaction with the joint opinion, the focal agent i adapts to the neighborhood. Suppose the stubbornness of i is ω_i and its openness is $1 - \omega_i$, then the adapted opinion $O_i^{Adapted}$ of agent i is calculated by Eq. 3. It represents a combination of relying on its original opinion and accepting a new opinion [8,16]

$$O_i^{Adapted} = O_i \times \omega_i + O_i^{Joint} \times (1 - \omega_i). \tag{3}$$

The interaction (opinion adaption) in our model, as shown in Eq. 3, can be understood as a Bayesian process: Stubbornness is viewed as a type of prior (the extent to which agents adhere to established beliefs); openness is viewed as a type of posterior (the degree to which agents adopt a new opinion). It is worth noting that in computer science, reinforcement learning is also frequently used to model agent interaction [23,57], while it is applied infrequently in the (computational) social science community [1,6]. The primary reason, in our opinion, is that the interaction in our model (and the vast majority of socially inspired models) is *not* an optimization process. Indeed, in the multi-agent computing community, agent interaction is primarily traced back to robot "foraging" or trial-and-error to escape the maze—which can be clearly viewed as an optimization process involving maximizing the cumulative (discounted) reward, in the majority of socially inspired models—including ours—opinion change occurs as a result of assimilation and adaptation to the neighborhood in society [1,19,33]. "Learning" typically refers to a Bayesian process, which is precisely the idea in the SCOOE model.

Intrinsic self-adjustments We have now seen how agents take advantage of extrinsic collective information within their incomplete neighborhoods. Agents also observe the local environment to adjust their opinions to seek a higher payoff. This is driven by intrinsic motivation. People sometimes engage in an activity just because they are drawn to do it [18,45].

We apply the imitation rule here that does not need direct interactions, transforms information in the population through observation and self-adjustment [43,50]. Again, a focal agent i only accesses a random partial neighborhood as its observation environment[1]. For these random neighbors, the focal agent i holds a probability W_{i,i_r} to imitate the local best-performing neighbor i_r (i.e., the neighbor with the highest cumulative payoff) by adopting i_r's opinion as its own opinion. The imitation probability W_{i,i_r} is expressed by Eq. 4 [50]

$$W_{i,i_r} = \frac{1}{1 + exp[(E_i - E_{i_r})/\mu]}. \tag{4}$$

E_i and E_{i_r} are cumulative payoffs of agent i and the local best-performing neighbor i_r. They reflect the long-term adaptation and assimilation into the society as stubborn agents with fairly different opinions will ultimately have a low cumulative payoff. μ is a noise parameter modeling irrational choices, and we set μ to $\mu = 1.5$ [50]. [2] μ allows the possibility to imitate opinions of agents with a lower cumulative payoff due to making an irrational choice. Agents observe the environment and keep a close eye on the cumulative payoff gap. They then adjust their opinions voluntarily without direct interactions to achieve a greater payoff and a better position in society.

In summary, we introduce (i) sparse opinion updates by taking incomplete information-based collective decision-making into account, and (ii) observation and self-adjustment of opinions without direct interaction with neighbors. Sparse interactions are achieved.

Open-ended structural dynamics

This section presents the open-ended structural dynamics with leaving and joining agents and the opinion-structure coevolution.

Leavers: At each time step, the agent with the lowest cumulative payoff leaves the society, which models an intention to exit a society where most individuals have fairly different positions (e.g., opinions). The stubbornness and openness of a leaver are recorded. All adjacent edges of this agent are removed from the society upon leaving. As society evolves, leaver-driven structural dynamics will demonstrate the confirmation bias more strongly, because stubborn agents with opinions fairly different from others will have a low payoff leading to their removal from the model. Opinion dynamics affect the cumulative payoff, influence which agents become leavers, and thus drive the structural coevolution of the system.

Joiners: At each time step, a newcomer v will join. As society evolves, the community structure constantly changes. Agent v has incomplete information about different communities. It detects the real-time community structure and attempts to join a random $community_v$ by connecting to randomly selected nodes within $community_v$. We assign a random opinion O_v to v and the recorded stubbornness/openness of

[1] These random partial neighbors for observation are different from the interaction neighborhoods as we allow more randomness. People also do not always consult with the same group within their social networks.

[2] When conducting long-term interactions, the cumulative payoff could reach several hundred. Thus, the value of noise is chosen to be slightly greater than the range [0,1] in previous work.

the leaver (see above) to v to keep the cognitive ability distribution stable within the society. Note that the cumulative payoff E_v of the incoming agent v is not comparable to that of existing agents when calculating the imitation probability and removing leavers, especially for long-term experiments (see Eq. 4). We accordingly assume that given v joining at time step t_v, agent v's initialized cumulative payoff E_v is adjusted by the corresponding payoff $R_v^{t_v}$ at time t_v multiplied by the number of completed interactions t_v. After initialization, the cumulative payoff E_v is calculated by regularly adding the corresponding payoff R_v^t at each time step t until v is removed or the system terminates globally.

After joining a community, the newcomer v chooses and connects to a node u in another community. We apply the preferential attachment principle (i.e., nodes with a higher connection degree have a stronger ability to attract new nodes added to the network), because "the rich getting richer" phenomenon is widely observed in real-world societies [5]. Thus, the probability $p_{v,u}$ for v choosing u to connect follows Eq. 5:

$$\forall u \in (G - community_v) : p_{u,v} \propto d_u / \sum d_u. \tag{5}$$

$G - community_v$ represents all of the other communities except for $community_v$, which the new node v joins. d_u represents the degree of node u. If only one community exists as the society evolves, the new node joins by connecting to only one node following preferential attachment. An algorithmic view of the SCOOE model is shown in Algorithm 1.

It is worth noting that opinion dynamics with sparse interactions provide criteria (i.e., cumulative payoff) for agents to leave society. Then, agents actively behave to drive structural evolution and opinion evolution. In the SCOOE model, we believe that both structural and opinion mechanisms contribute to the rich emergent dynamics observed in the experimental section.

Algorithm 1: The SCOOE Model

Initialize society, opinions, stubbornness, openness, payoffs, and neighbors;
for *each time step* $t (t = 1, \cdots, T)$ **do**
  //Opinion Dynamics with Sparse Interactions;
  for *each agent* i *in the society* **do**
    Agent i accesses partial random neighbors and generates a joint opinion O_i^{Joint} (see Eq. 1);
    Agent i receives the corresponding payoff R_i and updates the cumulative payoff E_i by O_i^{Joint} (see Eq. 2);
    Agent i is assimilated into the neighborhood with the adapted opinion $O_i^{Adapted}$ (see Eq. 3);
    Agent i voluntarily adjusts its opinion with an imitation probability W_{i,i_r} (see Eq. 4);
  end
  //The Coevolution of Open-ended Structure and Opinions;
  The agent with the lowest cumulative payoff leaves the society;
  Assign a random opinion O_v to a newcomer v;
  Assign the stubbornness/openness of the leaver to v;
  Adjust the cumulative payoff E_v of v;
  v detects the real-time community structure;
  if *multiple communities* (≥ 2) *can be found in the society* **then**
    v joins a random community by connecting to partial random nodes within this community;
    An edge is generated connecting v to a node u in another community, following the preferential attachment (see Eq. 5);
  else
    v connects to a node u in the society following the preferential attachment (see Eq. 5);
  end
end

Experiments

In this section, we describe our experiments and their results.

Experimental settings

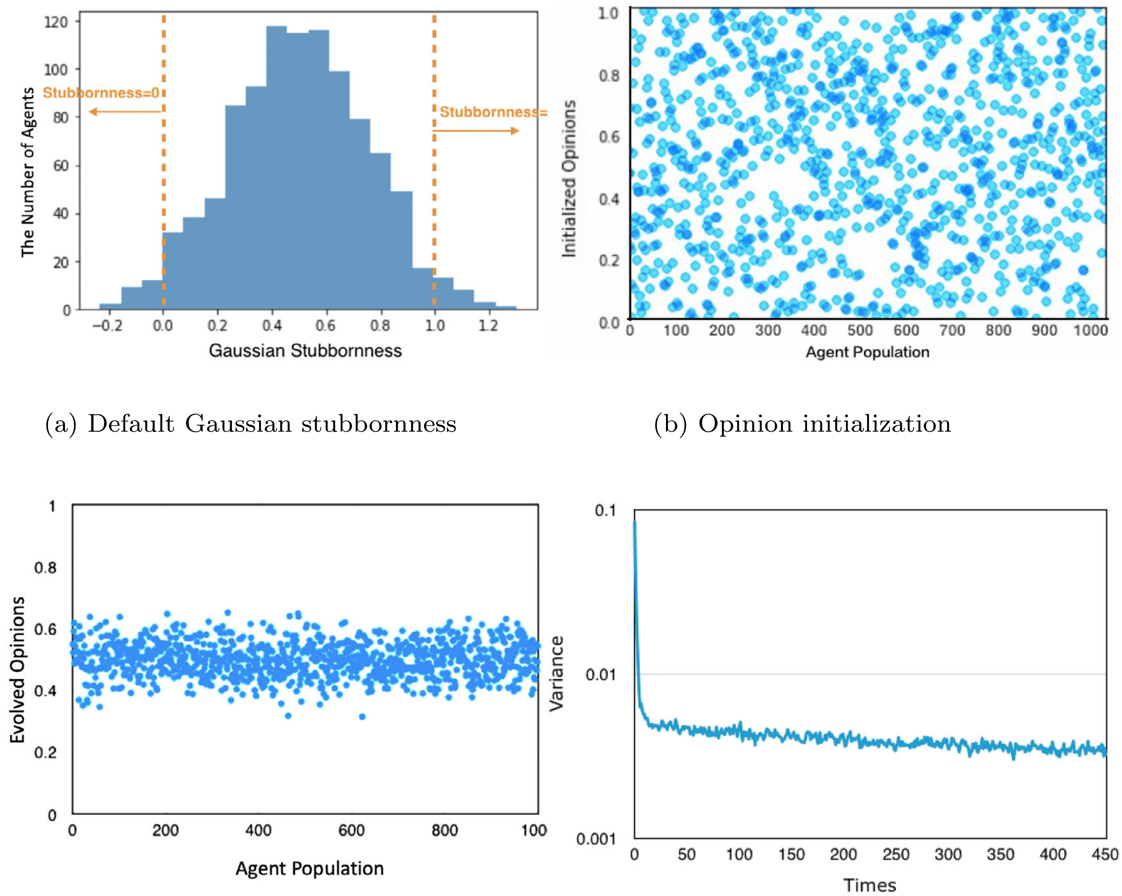
We create two small-world networks holding 500 nodes each to model two physically separated groups of people interacting to a certain degree. Therefore, randomly chosen edges connect the two small-world networks. We apply the Watts–Strogatz model to generate an individual small-world network [53]. Each node is connected to four nearest neighbors. The rewiring probability is set to 0.05. This structure constitutes the agent society, with each node representing an agent. A focal agent will only consider those agents connected by edges as neighbors and conduct sparse interactions based on the neighborhood. For initialization, we follow prior work from the psychology and computing realms [11,40] and set the stubbornness distribution to a Gaussian distribution with a mean of 0.5 and a standard deviation of 0.25. These parameters are chosen, so that most values lie between 0 and 1. In addition, we apply a cut-off, so that generated random numbers can only lie between 0 and 1, i.e., we constrain stubbornness to the interval between 0 to 1, as shown in Fig. 2a. Imitation noise is set to $\mu = 1.5$. The simulation is run for 450 Monte Carlo time steps.

Experimental results and analysis

Here, we report our experimental results by answering the following questions. We will explain and discuss the phenomena found in the discussion section.

How do group opinions evolve with sparse interactions?

We study how far the group opinions evolve away from their initial states, measured by the variance dynamics shown in



(a) Default Gaussian stubbornness

(b) Opinion initialization

(c) Evolved opinions under Gaussian stubbornness distribution

(d) Variance dynamics

Fig. 2 Opinion dynamics in the population with Gaussian stubbornness

Fig. 2. The opinions of agents are reasonably different at the start, because agents are assigned random opinions initially. As society evolves, we find two stages of evolution: a fast-decay phase (i.e., the variance of group opinions dramatically decreases from 0.084 to 0.005) and a slow-decrease phase (i.e., the variance slowly continues dropping to 0.003 at the end of the simulation). It is interesting to find that the final opinions are in a relatively narrow band and less polarized without firmly believing or strongly unbelieving the rumors among the agent population, even with some agents never changing their opinions (stubbornness =1) but being removed by the model. The Gaussian stubbornness distribution is also evenly distributed. The majority of the population keeps a balance between maintaining their original opinions and accepting a new opinion. Mirroring reality, we find that agents are more likely to stay open-minded to propagated news/rumors during *long-term* interactions in an *open-ended* society.

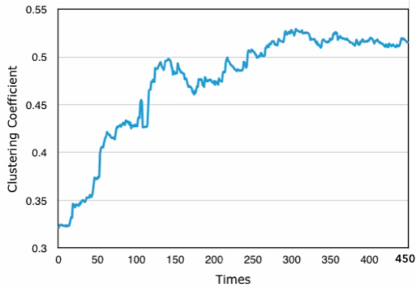
How do the opinion dynamics shape the structural co-dynamics?

For this question, we primarily focus on the dynamics of the clustering coefficient, average path length, degree distribution, and community structure. For real-time community detection, we use the most widely used method, namely the modularity-based method [10].[3]

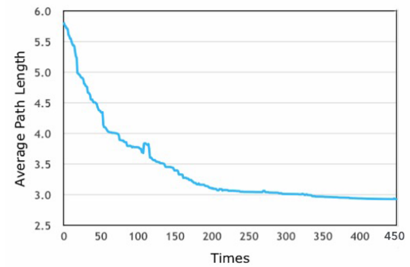
The society coevolves to be a holistically dense small-world structure with a heavy-tailed degree distribution

As shown in Figs. 3 and 4, we find an increase in the clustering coefficient and average degree, as well as a decrease in average path length and the number of communities. We initialize the society as two interconnected small-world networks. The random edges between them change the initialized small-

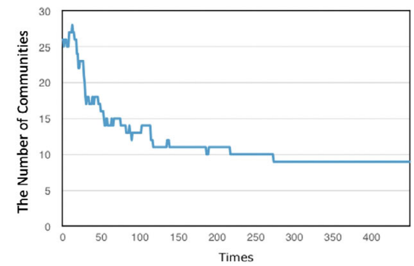
[3] We have compared several community detection methods. We find they do not strongly affect the results.



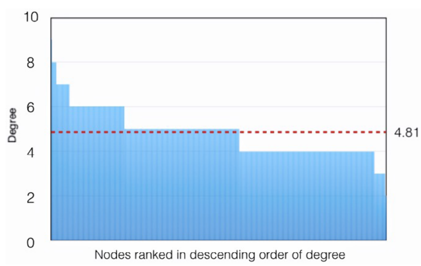
(a) Clustering coefficient



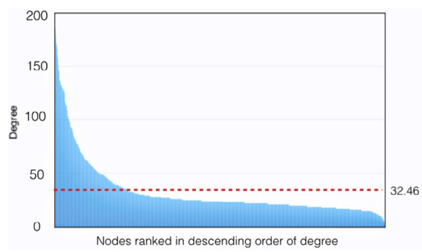
(b) Average path length



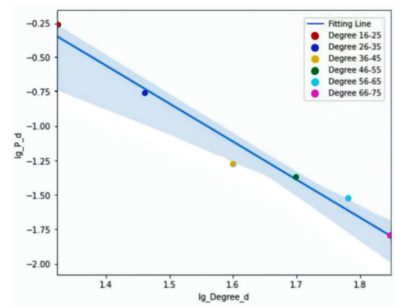
(c) The Number of communities



(d) Initialized degree distribution



(e) Coevolved degree distribution



(f) The relationships between node degree and node proportion

Fig. 3 Structural dynamics

world features by randomizing them to a certain degree. Thus, we find a chaotic society initially with 26 detected tiny communities and a coevolved society with nine segregated communities by the modularity-based method [10]. We also observe that the coevolved society has a small-world feature with a high clustering coefficient. It coevolves to be a more tightly knit group with dense connection degrees, high information transmission efficiency, and a low average path length due to network homophily. That is, the final opinions of the population are relatively consistent, leading to an increase in payoff and a decrease in conflicts (e.g., confirmation bias for fairly different opinions) in the game-playing upon interactions. This coevolutionary trend of the structure in turn boosts the evolution of a global opinion [39].

Although some small-world generation models, e.g., the Kleinberg model [28], do not generate heavy-tailed degree distributions, it is not surprising to find a heavy-tailed degree distribution appearing in the SCOOE model. The advantages of "the rich" become significant eventually because of preferentially added joiners. Specifically, we calculate the proportion $P(d)$ of nodes with connection degree d. We find that the relationships between node proportion $P(d)$ and node degree d can be approximated by a linear relationship $\log[P(d)] \propto (-\gamma) \times \log(d)$ with a negative slope $-\gamma \approx -2.758$ through linear regression within a 95% confidence interval. Note that the data points in Fig. 3f represent the average degree and the node proportion in different degree ranges. We only study the nodes in these degree ranges, because they fill most of the network. These nodes are enough to illustrate a linear relationship.

An emergent dialectic relationship between community segregation and cohesion

Cohesion is a concept of togetherness and connectedness among nodes within a network. There is no unified definition of cohesion, because it depends on the context. Previous literature has referred to it as cliques/communities, clusters,

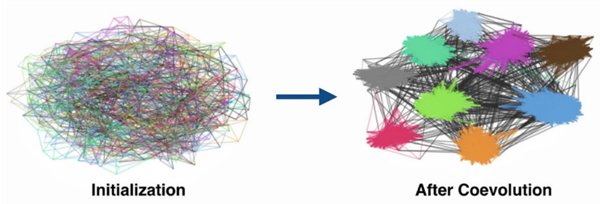


Fig. 4 The coevolved society after 450 time steps. The nodes and edges within a community are set with the same color. The color of external edges connecting two communities is set to black

or average degree [29].[4] Figure 4 shows our assessment of the community segregation and cohesion.

As shown in Fig. 4, the initialized society is desegregated and chaotic with a low level of cohesion (i.e., with a low average degree and clustering coefficient). As society coevolves, we find that it has a clear pattern of fewer segregated communities that become densely clustered (i.e., with a high average degree and clustering coefficient). Agents have disconnected social networks initially but highly cohesive social clusters eventually.

It is interesting to note that society becomes segregated but dense spontaneously and simultaneously with a global consensus and cohesion, but without multiple local-opinion "barycenters" that might emerge aligned with segregated communities [20]. Mirroring reality, as Neal et al. [41] suggest, a widely observed example in the real world is policy-making to reduce detrimental residential segregation. A widely adopted approach to introduce desegregated neighborhoods and reduce residential segregation is to improve cohesion, e.g., dense connections. However, a paradox exists between community segregation and cohesion. The society evolves to be dense with segregated communities, whereas a desegregated society is not as cohesive as we would expect.

What factors affect the evolved global opinion?

It is reasonable to suspect that the degree of stubbornness affects the emergence of a global opinion. Additionally, the SCOOE model incorporates multiple types of dynamics. What effect do these dynamics have on the evolution of a final opinion? This section will address these questions.

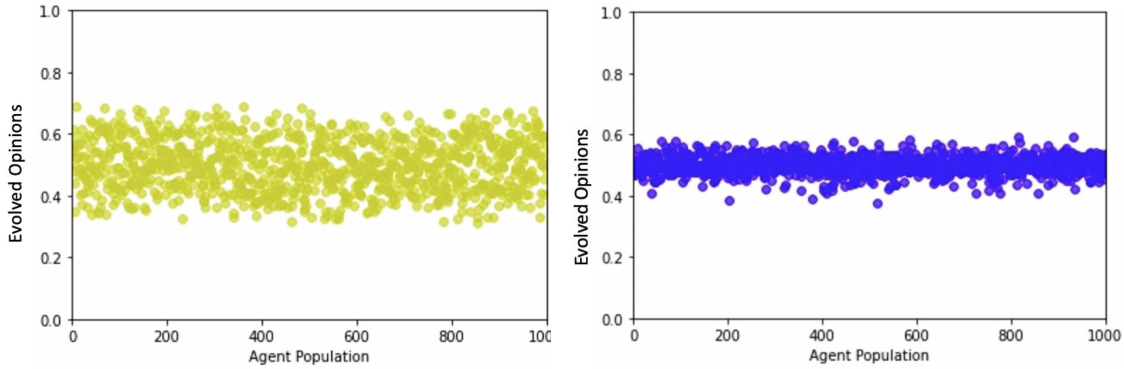
To study the influence of stubbornness distributions, we also test a Beta distribution and a Poisson distribution. We initialize the Beta distribution with two positive shape parameters $\alpha = 7$ and $\beta = 1$, and the Poisson distribution with the expected rate of occurrences $\lambda = 1$. We normalize the two generated distributions with the maximum value representing stubbornness $= 1$. The evolved opinions in these two cases are shown in Fig. 5. The variance comparison with different stubbornness distributions is shown in Fig. 6a. The variance comparison is defined as the ratio of the opinion variance for the Poisson/Beta stubbornness distributions to that for the baseline Gaussian stubbornness distribution at each time step t, $t \in [0, 450]$. The variance dynamics with different sparse interaction mechanisms in a population with a Gaussian stubbornness distribution are shown in Fig. 6b.

Stubbornness is generally small in a population with Poisson stubbornness. Agents are very flexible to become followers of the propagated news/rumor. As a result, it will be easier to pass the fast-decay phase, and we observe an initial lower variance than the baseline shown in Fig. 6a. Because of the flexibility in updating opinions, evolved opinions are still inconsistent at the end of 450 time steps, and the final variance is relatively large. In contrast, the agent population generally has much higher Beta stubbornness. Accordingly, we find an initial increase in the variance ratio to pass the fast-decay phase shown in Fig. 6a. Because of the high stubbornness, final opinions are stable with few changes, and lower final variance than the baseline can be observed.

It is challenging to drive the global opinion evolution among a stubborn population, e.g., the initially weak emergence of the global opinion in the population with high Beta stubbornness. However, it is interesting to find the most unified global consensus in such a society with many agents only weakly changing opinions. This unusual phenomenon is due to the open-endedness of society. The most stubborn agents will be considered maladapted to the environment and removed as society evolves. Agents will be assimilated by agents who surround them. No matter the initial opinions they hold in stubborn crowds, they will finally have a relatively unified group consensus after the *long-term* interactions and the slow assimilation of opinions crowding out dissidents in an *open-ended* society. We can say that these high stubbornness values serve as a "wall"—newcomers with similar opinions will be accepted, while newcomers with opinions out of this range will be removed quickly.

We additionally test the model without the intrinsic self-adjustment mechanism, as shown in Fig. 6b. A widely studied contagion phenomenon in social networks is that the chance to adopt a contested "innovation" (e.g., firmly believing a piece of news/rumor) will be smaller for an individual with more neighbors [8,21]. When a focal agent aggregates the joint opinion by collective decision-making, extreme opinions (e.g., a strong endorsement) of selected neighbors are neutralized by weighted averaging. This effect will be more significant for high-degree nodes, given the larger share of their neighbors. On the other hand, high-degree nodes with

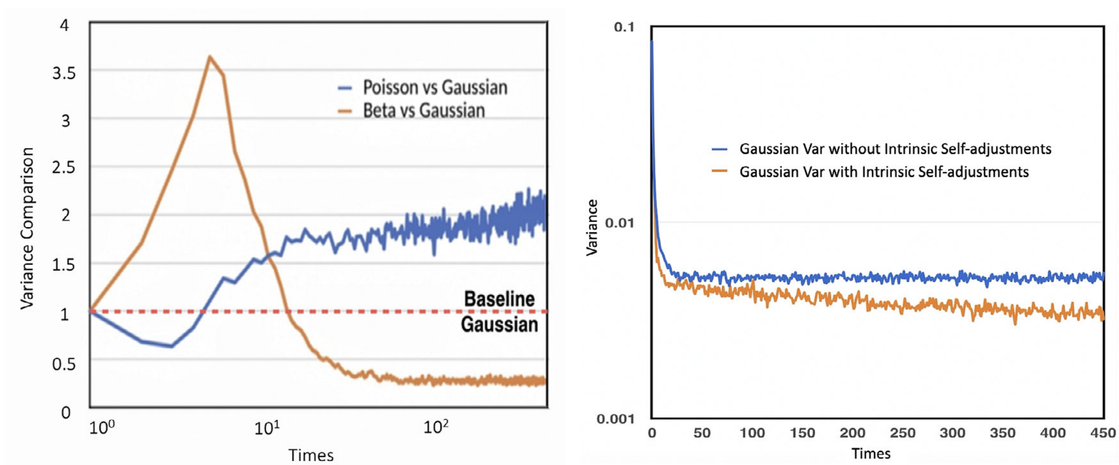
[4] Some work has also applied the k-component for measuring network cohesion. The k-component of a network is the maximum sub-graph in which we need to remove at least k nodes to break this sub-graph into more components. We do not study the k-component here, because the "giant component" that fills most of the network is always found in an undirected network, while the rest of the network is divided into many scattered small components [31]. This does not help us understand cohesion and segregation but works for connectivity.



(a) Poisson stubbornness distribution

(b) Beta stubbornness distribution

Fig. 5 The evolved opinions with different stubbornness distributions



(a) Variance comparison with different stubbornness distributions

(b) The dynamics of the variance of opinions with/without self-adjustment mechanism

Fig. 6 Opinion dynamics with different stubbornness distributions and sparse interaction mechanisms

a fewer likelihood of being extreme have a more substantial impact on the weighted aggregation method and a more extensive influence range. At the same time, collective interactions decrease the probability of interacting directly with extreme agents and being affected by them. Therefore, the extrinsic collective interaction mechanism boosts the emergence of a global consensus, as shown in the similar trends of the fast-decay phase in the two cases in Fig. 6b. It plays fewer roles when the population rapidly reaches a pre-consensus (the start of the slow-decrease phase in variance dynamics), given the constantly adapted local interaction environment with the randomness to select neighbors, the joiners/leavers, and a constant injection of new opinions. The intrinsic adjustment mechanism continues to further the emergence of a global consensus and weakens conflicts by direct imitation. It can be said that extrinsic collective interactions primarily play a role in the fast-decay phase of the variance dynamics, whereas intrinsic adjustments mainly play a role in the slow-decrease phase. Their interplay works to enhance the evolution of a global opinion. Note that when we set the self-adjustment noise μ to a very large value, we can observe similar results to the case of removing the self-adjustment mechanism.

Discussion

It is crucial to design simple but practical agent-based models linked to the phenomena of interest. This section revisits and discusses the proposed mechanisms by focusing on their effects on the consensus evolution within groups.

Lean and fast decision strategies with incomplete information

A broad assumption in the widely cited bounded-confidence model is that rational agents owning the perfect information poll their neighborhood and select neighbors to interact only if their opinions are sufficiently close to their own. This assumption facilitates polarization and global conflicts [20]. It has been widely recognized that it is difficult to evolve a global consensus for large population sizes [26,30,56], because multiple local consensuses might be distributed in a society. As a result, such a system needs more bottom-level interactions to pass the formation of these local consensuses. Our results validate several earlier findings with different mechanisms and remarkably boost the evolution even in a stubborn population [35,46,54]. Unlike some bounded-confidence models, e.g., [12,20], here, we start by assuming that bounded-rational agents only access a partial neighborhood (incomplete information) to aggregate a joint opinion. Confirmation bias is represented by the stipulation that adopting more similar opinions will bring a higher payoff. We find that conflicts among bounded-rational agents are weakened globally and rapidly. Bounded rationality with incomplete information forms lean and fast decision strategies to reduce conflicts under uncertainties, whereas complete information weakens group coordination, as suggested by some literature from psychology [17].

Open-endedness enables permanently novel opinions

We find that eventually evolved opinions are wholly unified in some closed-society models [46]. The continuous addition and removal of agents and the structure/opinion dynamics they bring with them influence neighbors and neighbors' surroundings in a cascading fashion. Though the designed mechanisms strongly facilitate the evolution, it is impossible to reach a highly unified global consensus. One can only approach it no matter whether the randomness or noise exists, as the slow-decrease phase in variance dynamics shown in Figs. 2 and 6. It can also be said that the SCOOE model is robust to boost and enhance the evolution of a global opinion as it successfully defends against the interference of a constant injection of novel opinions.

The interplay between sparse interaction and open-ended structure reduces the echo chamber effect

The echo chamber effect in social media studies describes a situation where local opinions are reinforced by repetition inside a closed society and insulated from rebuttal or different opinions (confirmation bias). Surprisingly, a substantial body of research indicates that people are not as polarized as we would expect in the echo chamber, both empirically [2,4,25,47] and theoretically [35,46,54]. We offer two possible theoretical justifications for this apparent discrepancy between evidence and intuition: From the perspective of opinion dynamics, the focal agent considers the collective opinion based on a limited view of the neighborhood, which reduces polarization quickly, as discussed in "Lean and fast decision strategies with incomplete information" and "What Factors Affect the Evolved Global Opinion?". From a structural dynamics perspective, the society in our model (and also in the real world) is open-ended and constantly changing. It imparts persistent dynamics on the neighborhood structure, resulting in neighbors with whom the focal agent interacts being neither isolated nor static. When we examine previous models based on a closed structure, some work has shown global/local polarization and extreme opinions [3,23,38]. The open-endedness feature with a constant injection of novel opinions in the SCOOE model helps a population defend against the echo chamber effect and stay open-minded. It reduces the chances of extreme results, because extreme agents are likely to be removed from society. It also mirrors the findings of a global consensus formation in a population with high Beta stubbornness. In general, we believe that opinion and structural mechanisms are inextricably linked and that their interplay helps reduce polarization.

Conclusions

In this paper, we propose the SCOOE model, a coevolutionary opinion dynamics model with sparse interactions in an open-ended society. Two-phase evolution shows that the extrinsic collective interaction mechanism boosts the evolution; the intrinsic adjustment mechanism slowly reduces conflicts. Their interplay facilitates the effective and robust formation of a global opinion. The model also provides a new direction for small-world network generation from social complexity and interaction perspectives. As the model evolves, agents tend to be connected closer to others. The agent society shows a small-world characteristic with a heavy-tailed degree distribution and emerges to become more segregated and cohesive. Different emergent trends can be found in flexible, normal, and stubborn populations, with a link to a potential explanation in cognitive science and psychology. We expect to apply the SCOOE model in a broader field, e.g., media and communication studies and decentralized control of asynchronous systems.

Though the proposed mechanisms show promise in experiments, there are some open questions. While we successfully introduced a limited view of focal agents, payoff information is accessible to all agents. We are interested in a model with even less information. Is it possible to design an incomplete information model with hidden payoff information when agents adjust the opinions according to the environmental feedback? An empirical study based on real-world data to calibrate the model and a further relaxation of assumptions to make the model even simpler are also valuable directions in the future.

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Declarations

Conflict of interest The authors declare that there is no conflict of interest.

Code is available Code is available in the supplementary materials.

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