Unit 3: Time Response, Part 3: Systems with Additional Poles or Zeroes

Engineering 5821:
Control Systems I

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Systems with Additional Poles or Zeroes

The formulas for T_p, T_s, etc... are specific to second-order systems. However, sometimes systems with additional poles or zeroes are well approximated as second-order.

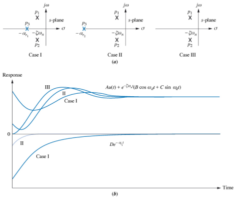
Consider the addition of a third pole to a two-pole system. First, note that the only way to add exactly one more pole is for the pole to be on the real-axis.

The system response is as follows,

$$C(s) = \frac{bc}{s(s^2 + as + b)(s + c)} = \frac{A}{s} + \frac{Bs + C}{s^2 + as + b} + \frac{D}{s + c}$$

In the TD, the third term adds De^{-ct} to the second-order step response. What effect does this have?

Consider the three following cases for the position of the third pole:



If the third pole is near the other two then its effect is strong. However, as it is moved to the left it decays at a faster rate and therefore has less effect on the overall signal.

Even if the exponential decays quickly, we may still be concerned about its initial effect. Actually, the effect of the third pole decreases in magnitude (as well as decay rate) as the pole moves to the left. Consider again the system response:

$$C(s) = \frac{bc}{s(s^2 + as + b)(s + c)} = \frac{A}{s} + \frac{Bs + C}{s^2 + as + b} + \frac{D}{s + c}$$

We can evaluate these constants:

$$A = 1 \qquad B = \frac{ca - c^2}{c^2 + b - ca}$$
$$C = \frac{ca^2 - c^2a - bc}{c^2 + b - ca} \qquad D = \frac{-b}{c^2 + b - ca}$$

As $c \to \infty$ $D \to 0$. Thus, for a third-pole that is far to the left of the dominant poles, the system response is approximately second-order. How far is far enough to discount this third pole? **Rule of thumb:** If the pole's real part is five times the real part of the dominant poles, then the system can be approximated as second-order.

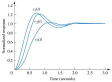
e.g. Which of T_2 or T_3 is a better approximation to T_1:

$$T_1(s) = \frac{24.542}{s^2 + 4s + 24.542}$$
$$T_2(s) = \frac{24.542}{(s+10)(s^2 + 4s + 24.542)}$$
$$T_3(s) = \frac{24.542}{(s+3)(s^2 + 4s + 24.542)}$$

The step responses for T_2 and T_3 have the following form:

$$c_i(t) = 1 + k_1 e^{p_i t} + k_2 e^{-2t}\cos(4.532t + \phi)$$

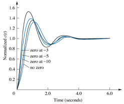
Solution: The pole for T_2 is 5 times the real component of the dominant poles and can therefore be approximately neglected.



Additional Zeros

What is the effect of adding a zero to a second-order system? The addition of zeros affect the response, but the form of the response is preserved (it will still be an exponentially damped sinusoid).

Consider adding a real-axis zero to the left-plane. We begin with a second-order system with poles at $-1 \pm j2.828$.



It appears that the closer the zero is to the dominant poles, the more impact it has.

Consider the partial-fraction expansion:

$$\begin{aligned} T(s) = \frac{s+a}{(s+b)(s+c)} &= \frac{A}{s+b} + \frac{B}{s+c} \\ &= \frac{(-b+a)/(-b+c)}{s+b} + \frac{(-c+a)/(-c+b)}{s+c} \end{aligned}$$

What if a is large relative to b and c?

$$T(s) \approx a\left[\frac{1/(-b+c)}{s+b} + \frac{1/(-c+b)}{s+c}\right] = \frac{a}{(s+b)(s+c)}$$

In this case a acts as a simple gain factor, which does not alter the form of the response.

This can be illustrated in another way. Our original system's response is as follows:

$$C(s) = R(s)G(s)$$

Assume the system is now modified such that $G'(s) = (s+a)G(s)$. The new system response will be,

$$\begin{aligned} C'(s) &= R(s)G'(s) = R(s)(s+a)G(s) \\ &= (s+a)C(s) = sC(s) + aC(s) \end{aligned}$$

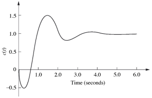
The response is composed of the derivative of the original response plus a scaled version of the original response.

$$C'(s) = sC(s) + aC(s)$$

The derivative of the second order response is initially positive which means that the zero affects the response by increasing overshoot.

SIMULINK DEMO

If a is negative then the initial response may be negative, causing the system to initially move in the opposite direction:



Such a system is known as a **nonminimum-phase system**.

Pole-Zero Cancellation

We can cancel poles and zeros even when they are not exactly equal. Consider the following system:

$$\begin{aligned} C_1(s) &= \frac{26.25(s+4)}{s(s+3.5)(s+5)(s+6)} \\ &= \frac{1}{s} - \frac{3.5}{s+5} + \frac{3.5}{s+6} - \frac{1}{s+3.5} \end{aligned}$$

The residue of the pole at 3.5 is not negligible compared to the other residues. We cannot accurately approximate this system as second-order. However, we can for the following system:

$$\begin{aligned} C_2(s) &= \frac{26.25(s+4)}{s(s+4.01)(s+5)(s+6)} \\ &= \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} - \frac{0.033}{s+4.01} \\ &\approx \frac{0.87}{s} - \frac{5.3}{s+5} + \frac{4.4}{s+6} \end{aligned}$$