Unit 3: Time Response
Part 1: Poles and Zeros and First-Order Systems

Engineering 5821:
Control Systems I

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Poles and Zeros

This unit of the course considers the relationship between a system's transfer function and the response of the system in the time domain. It corresponds to chapter 4 in the textbook.

The poles and zeros of the input and the transfer function can be quickly inspected to determine the form of the system response. Of course, we can obtain this form from the ILT, but looking at the poles and zeros allow us to see it more *quickly*.

Definitions

The **poles** of a function are values of s for which the function becomes infinite.

e.g. $s = -2$ is the pole of $\frac{1}{s+2}$.

The **zeros** of a function are values of s for which the function becomes zero.

e.g. $s = -3$ is the zero of $\frac{s+3}{s+2}$.

Sometimes we also classify as zeros or poles roots of the denominator (poles) or numerator (zeros) which are common and can therefore be cancelled. These so-called zeros or poles lack the ability to make the function go to zero or infinity, yet are sometimes referred to as zeros or poles nevertheless.

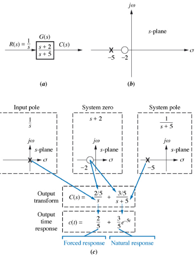
e.g. Consider the system with transfer function $G(s) = \frac{s+2}{s+5}$. The input is the unit step function. The system response is

$$C(s) = \frac{1}{s}\frac{s+2}{s+5} = \frac{A}{s} + \frac{B}{s+5} = \frac{2/5}{s} + \frac{3/5}{s+5}$$

Applying the ILT we obtain the time-domain response

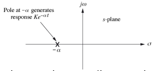
$$c(t) = \frac{2}{5} + \frac{3}{5}e^{-5t}$$

The following figure illustrates the relationship between the system response and the poles and zeros...



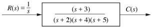
Notice the following:

- The input pole generates the form of the *forced response*
- The system pole generates the form of the *natural response*
- This pole is on the negative real axis. Hence, it generates a decaying exponential



- The zeros and poles together contribute to the calculation of A and B

e.g. What is the form of the system response for the following system?



First-Order Systems

Consider the step respone of a first-order system without zeros:

$$C(s) = R(s)G(s) = \frac{a}{s(s+a)}$$

Applying partial fractions and the ILT, we obtain,

$$c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$$

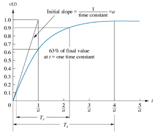
The input pole (at $s = 0$) generates the forced response $c_f(t) = 1$ while the system pole yields the natural response $c_n(t) = -e^{-at}$.

We can rewrite this function as follows,

$$\begin{aligned} c(t) &= 1 - e^{-at} \\ &= 1 - e^{-t/\tau} \end{aligned}$$

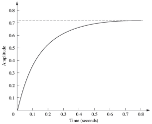
where $\tau = \frac{1}{a}$ is the **time constant**. At the time constant the function reaches 63% of its final value.

$$c(1/a) = 1 - e^{-1} = 1 - 0.37 = 0.63$$



Sometimes we will know the form the response but will be required to derive the parameters experimentally. e.g. Assume the step response has the following form and time-domain plot:

$$C(s) = \frac{K}{s(s+a)} = \frac{K}{a}\frac{a}{s(s+a)}$$



Determine a and K.

The asymptote yields an estimate of $K/a \approx 0.72$. 63% of this is 0.45 which is reached around $t = 0.15$. Hence $a = 1/0.15 = 6.67$ and $K = 4.8$.

In addition to the time constant, there are other measures of first-order system performance that are often used,

- Rise time: The time for the response to go from 10% to 90% of its final value
$$T_r = \frac{2.2}{a}$$
- Settling time: The time for the response to reach and stay within 2% of its final value
$$T_s = \frac{4}{a}$$