Supplementary Material: The Rotation Matrix

Computer Science 4766/6912

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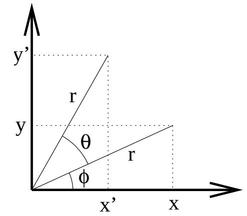
- Assume we have a vertex at (x, y) which is to be rotated counterclockwise about the origin by an angle θ
- In polar coordinates, this vertex is at (r, ϕ); We can express this in Cartesian coordinates:

$$\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \end{aligned}$$

- Now the rotation by θ can be understood as an addition of angles:



- The coordinates of the rotated vertex are as follows:

$$\begin{aligned} x' &= r\cos(\theta + \phi) \\ y' &= r\sin(\theta + \phi) \end{aligned}$$

- We can now make use of the following trigonometric identities:

$$\begin{aligned} \cos(a + b) &= \cos a \cos b - \sin a \sin b \\ \sin(a + b) &= \sin a \cos b + \cos a \sin b \end{aligned}$$

- After a few steps (COVERED ON BOARD) we obtain,

$$\begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= x\sin\theta + y\cos\theta \end{aligned}$$

- In vector form, this can be written as:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- or $\boldsymbol{v}' = \boldsymbol{R_{ccw}}(\theta)\boldsymbol{v}$

  where $\boldsymbol{v}$ is the original vertex, $\boldsymbol{v}'$ is the rotated vertex, and $\boldsymbol{R_{ccw}}$ is the counter-clockwise (hence 'ccw') rotation matrix
- **Note:** If the direction of rotation is not specified, then assume counter-clockwise. In other words:

$$\boldsymbol{R}(\theta) = \boldsymbol{R_{ccw}}(\theta)$$