Supplementary Material:
The Rotation Matrix

Computer Science 6912
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Assume we have a vertex at \((x, y)\) which is to be rotated counterclockwise about the origin by an angle \(\theta\). In polar coordinates, this vertex is at \((r, \phi)\); we can express this in Cartesian coordinates:

\[
\begin{align*}
x &= r \cos \phi \\
y &= r \sin \phi
\end{align*}
\]

Now the rotation by \(\theta\) can be understood as an addition of angles:

\[
\begin{align*}
x' &= r \cos (\theta + \phi) \\
y' &= r \sin (\theta + \phi)
\end{align*}
\]

We can now make use of the following trigonometric identities:

\[
\begin{align*}
\cos (a + b) &= \cos a \cos b - \sin a \sin b \\
\sin (a + b) &= \sin a \cos b + \cos a \sin b
\end{align*}
\]

After a few steps (COVERED ON BOARD) we obtain,

\[
\begin{align*}
x' &= x \cos \theta - y \sin \theta \\
y' &= x \sin \theta + y \cos \theta
\end{align*}
\]

In vector form, this can be written as:

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = 
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]

or \(v' = R_{ccw}(\theta)v\)

where \(v\) is the original vertex, \(v'\) is the rotated vertex, and \(R_{ccw}\) is the counter-clockwise (hence ‘ccw’) rotation matrix.

**Note:** If the direction of rotation is not specified, then assume counter-clockwise. In other words:

\[
R(\theta) = R_{ccw}(\theta)
\]