Supplementary Material: The Rotation Matrix

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

May 23, 2018
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    x &= r \cos \phi \\
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Now the rotation by \(\theta\) can be understood as an addition of angles:
The coordinates of the rotated vertex are as follows:

\[ x' = r \cos(\theta + \phi) \]
\[ y' = r \sin(\theta + \phi) \]

We can now make use of the following trigonometric identities:

\[ \cos(a + b) = \cos a \cos b - \sin a \sin b \]
\[ \sin(a + b) = \sin a \cos b + \cos a \sin b \]

After a few steps (COVERED ON BOARD) we obtain,

\[ x' = x \cos \theta - y \sin \theta \]
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After a few steps (COVERED ON BOARD) we obtain,

\[ x' = x \cos \theta - y \sin \theta \]
\[ y' = x \sin \theta + y \cos \theta \]
In vector form, this can be written as:

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\begin{bmatrix}
x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]

or

\[
v' = R_{ccw}(\theta)v
\]

where \(v\) is the original vertex, \(v'\) is the rotated vertex, and \(R_{ccw}\) is the counter-clockwise (hence 'ccw') rotation matrix.

Note: If the direction of rotation is not specified, then assume counter-clockwise. In other words:

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