Supplementary Material: The Rotation Matrix

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

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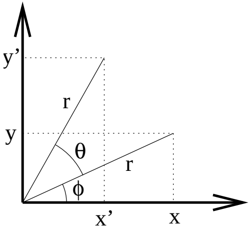

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