Planning: Part 2
Probabilistic Planning

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

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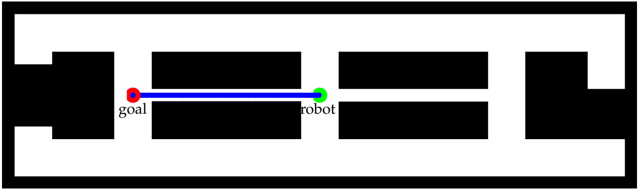

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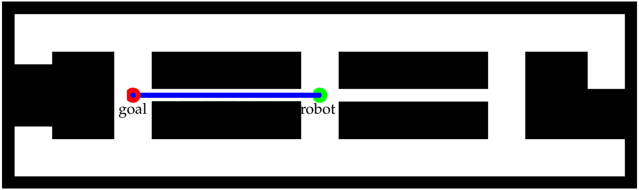

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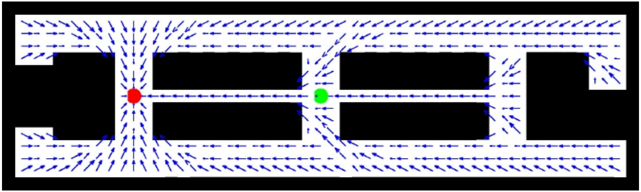

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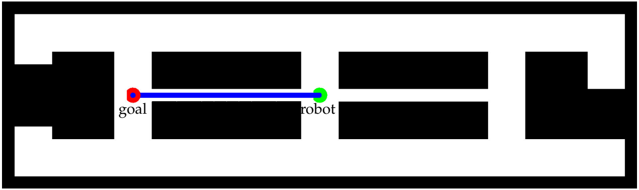

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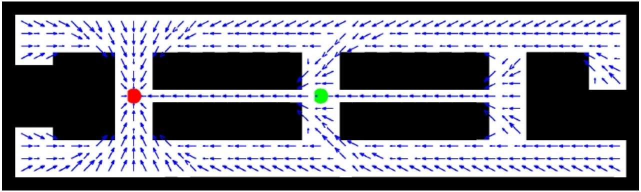

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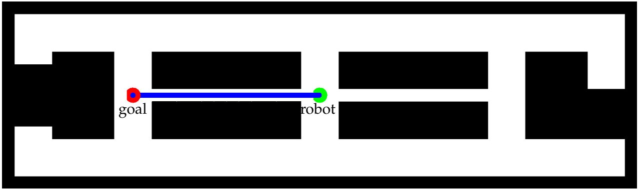

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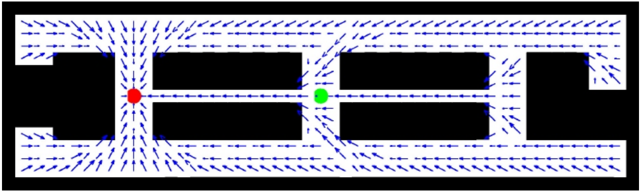

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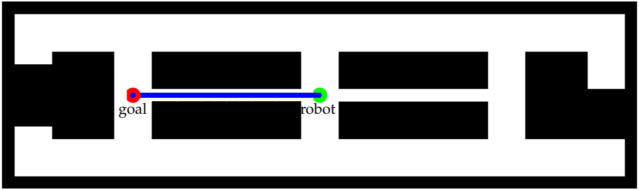

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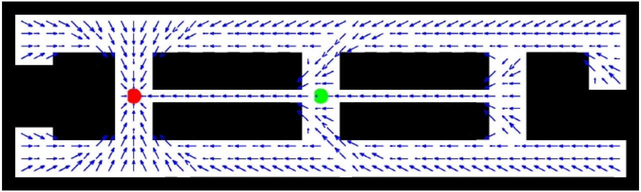

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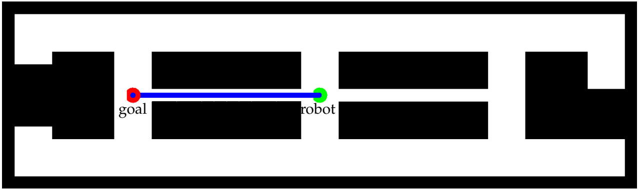

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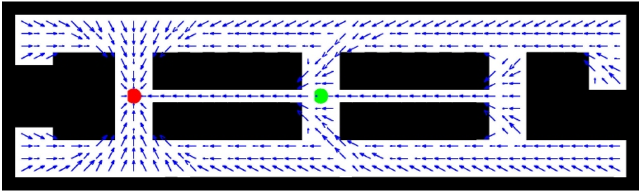

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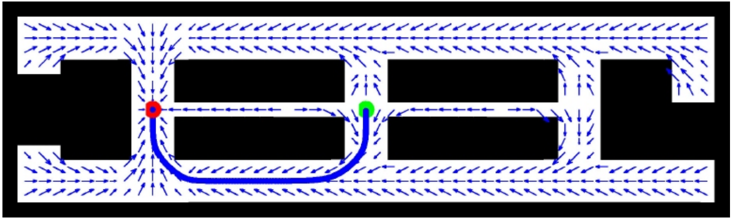

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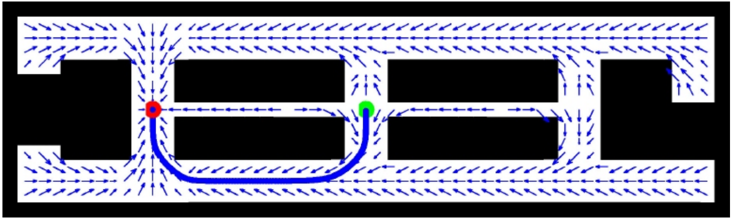

Notice that the longer path is now preferred since it reduces the risk of running into a wall.

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We get a reward of 100 for reaching the goal state and a movement penalty of -1. We attempt to maximize the expected future payoff, which will hopefully lead us to the goal state while minimizing future penalties.

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The value iteration algorithm uses $T = \infty$ with future payoffs discounted by γ^τ.

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The discount factor γ is applied to the immediate reward $r(x,u)$ as before, as well as being applied within the definition of $V_1(x)$.

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Since $V_T(x)$ is defined recursively, you can see that the discount factor will be applied T times. γ is usually set to a value just less than one (e.g. 0.99). So γ^T will become vanishingly small as T increases. Thus, we expect $V_T(x)$ to converge to some finite value as $T \to \infty$.

We can infer the general form for the optimal policy and associated value function for any T,

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\pi_T(x) &= \underset{u}{\operatorname{argmax}} \left[r(x,u) + \int V_{T-1}(x') p(x'|u,x) dx' \right] \\
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Algorithm (VERSION 1)

discreteValueIteration()

for i = 1 to N

$V(x_i) = r_{min}$

end for

repeat until convergence

for i = 1 to N

$$V(x_i) = \gamma \max_u \left[r(x_i, u) + \sum_{j=1}^{N} V(x_j) p(x_j | u, x_i) \right]$$

end for

end repeat

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end repeat

controlPolicy(x_i, V)

return $\underset{u}{\operatorname{argmax}} \left[r(x_i, u) + \sum_{j=1}^{N} V(x_j) p(x_j | u, x_i) \right]$

There is an issue with this version (VERSION 1) of value iteration.
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Using this, and the similarly modified policy rule, gives us VERSION 2 of the algorithm...

Algorithm (VERSION 2)

discreteValueIteration()

for i = 1 to N

$V(x_i) = r_{min}$

end for

repeat until convergence

for i = 1 to N

$$V(x_i) = \gamma \max_u \left[\sum_{j=1}^{N} p(x_j | u, x_i) \left(r(x_j, u) + V(x_j) \right) \right]$$

end for

end repeat

Algorithm (VERSION 2)

discreteValueIteration()

for i = 1 to N

$V(x_i) = r_{min}$

end for

repeat until convergence

for i = 1 to N

$$V(x_i) = \gamma \max_u \left[\sum_{j=1}^{N} p(x_j | u, x_i) \left(r(x_j, u) + V(x_j) \right) \right]$$

end for

end repeat

controlPolicy(x_i, V)

return $\underset{u}{\operatorname{argmax}} \left[\sum_{j=1}^{N} p(x_j | u, x_i) \left(r(x_j, u) + V(x_j) \right) \right]$

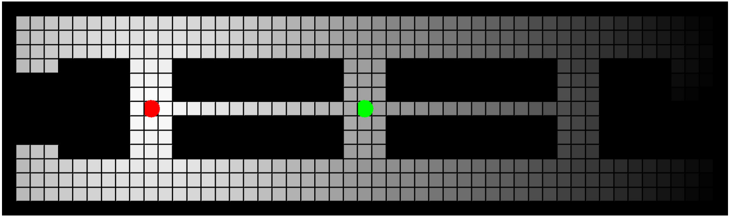


Figure 14.4 An example of an infinite-horizon value function T_∞, assuming that the goal state is an "absorbing state." This value function induces the policy shown in Figure 14.2a.

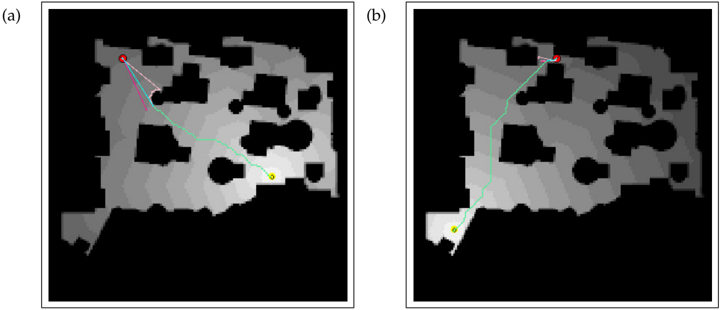


Figure 14.5 Example of value iteration over state spaces in robot motion. Obstacles are shown in black. The value function is indicated by the gray shaded area. Greedy action selection with respect to the value function lead to optimal control, assuming that the robot's pose is observable. Also shown in the diagrams are example paths obtained by following the greedy policy.

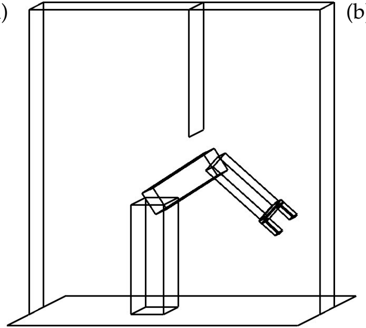
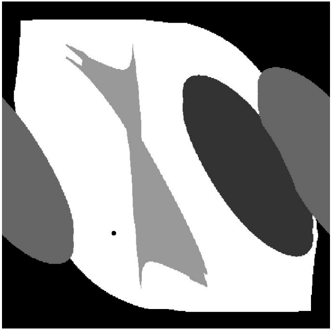
(a)  (b) 

Figure 14.6 (a) 2-DOF robot arm in an environment with obstacles. (b) The *configuration space* of this arm: the horizontal axis corresponds to the shoulder joint, and the vertical axis to its elbow joint configuration. Obstacles are shown in gray. The small dot in this diagram corresponds to the configuration on the left.

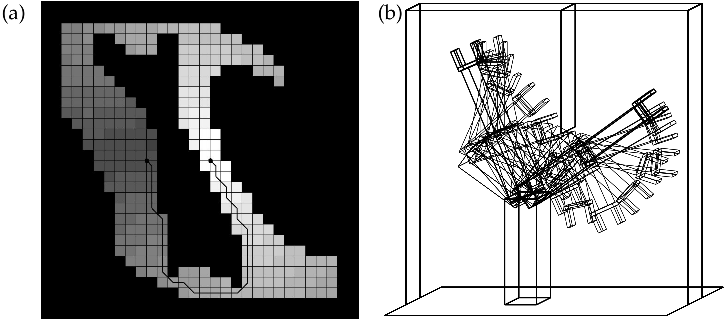


Figure 14.7 (a) Value iteration applied to a coarse discretization of the configuration space. (b) Path in workspace coordinates. The robot indeed avoids the vertical obstacle.