Planning: Part 1
Classical Planning

Computer Science 4766/6912

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- Mobile robots have much lower degrees-of-freedom.
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Shaded positions in configuration space indicate that the robot would intersect objects in its workspace.
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  - We can account for the reduction of the robot to a point by *inflating* all obstacles by the robot’s actual radius.

We will briefly describe three general approaches to planning:

1. Road maps: Identify a set of routes within the free space.
2. Cell decomposition: Discriminate between free and occupied cells.
3. Potential fields: A potential function attracts the robot to the goal, while repelling it from obstacles.

The first two actually just describe how to decompose space into a graph. Once the graph is obtained, a shortest path algorithm (e.g., Dijkstra, A* 

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Visibility graphs are easy to implement and generate optimal (shortest possible length) paths. However, these paths skirt the edges of obstacles, possibly jeopardizing the robot. A generalized Voronoi diagram (GVD) consists of all points in free space which are equidistant to the two closest obstacles. Paths are safer, but longer, than those of visibility graphs. A robot not on the GVD can easily join it by moving away from the nearest obstacle until the GVD is reached.
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Left: Obstacle map
Centre: Potential field
Right: Gradient contour and path followed
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