Planning: Part 1
Classical Planning

Computer Science 4766/6912

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Memorial University of Newfoundland

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  - Mobile robots have much lower degrees-of-freedom

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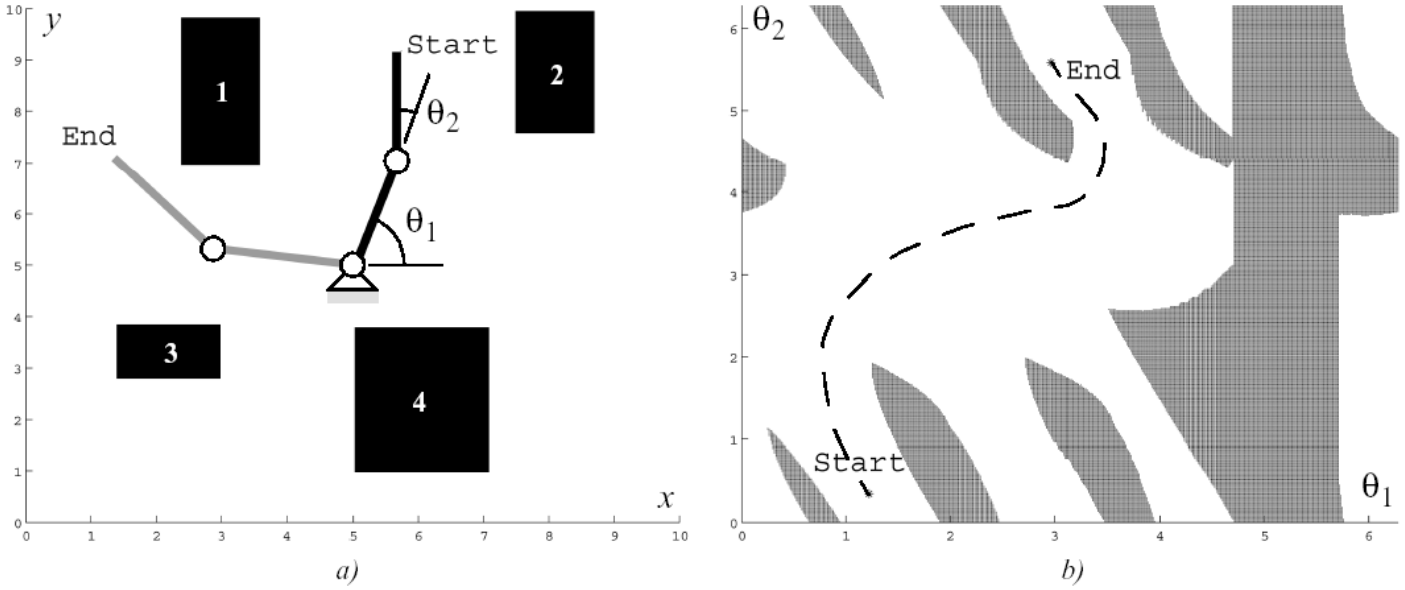

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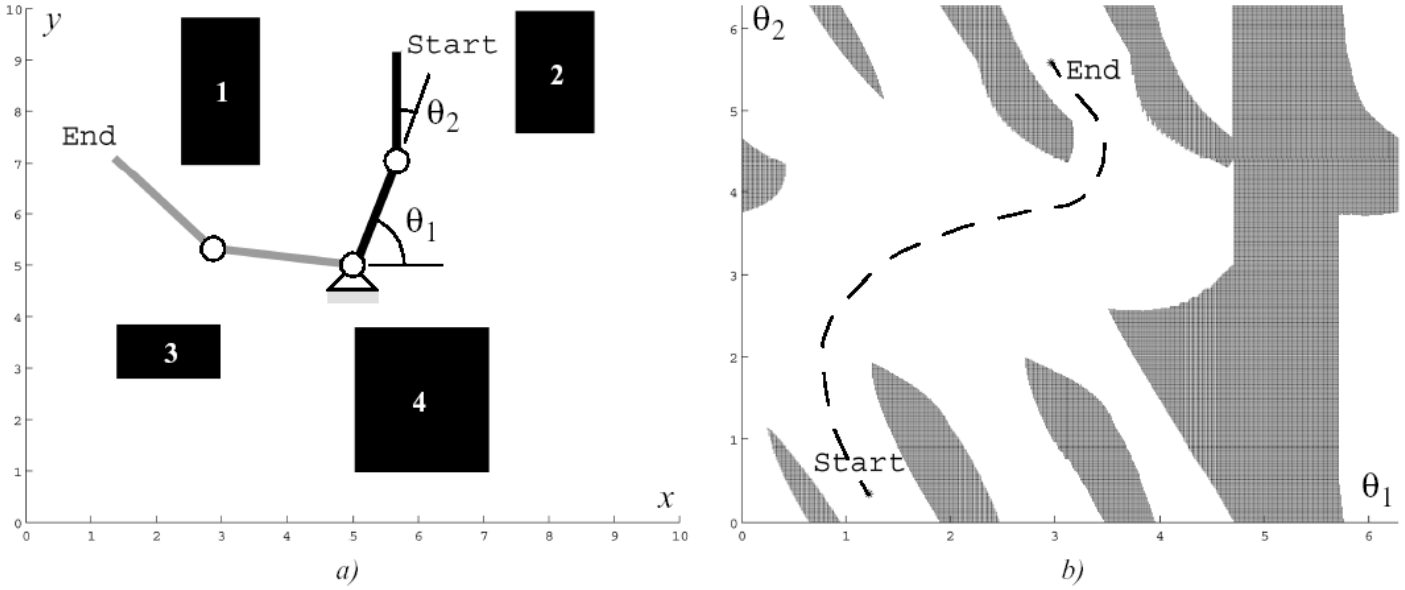

Shaded positions in configuration space indicate that the robot would intersect objects in its workspace.

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- The first two actually just describe how to decompose space into a graph. Once the graph is obtained, a shortest path algorithm (e.g. Dijkstra, A*) is applied.

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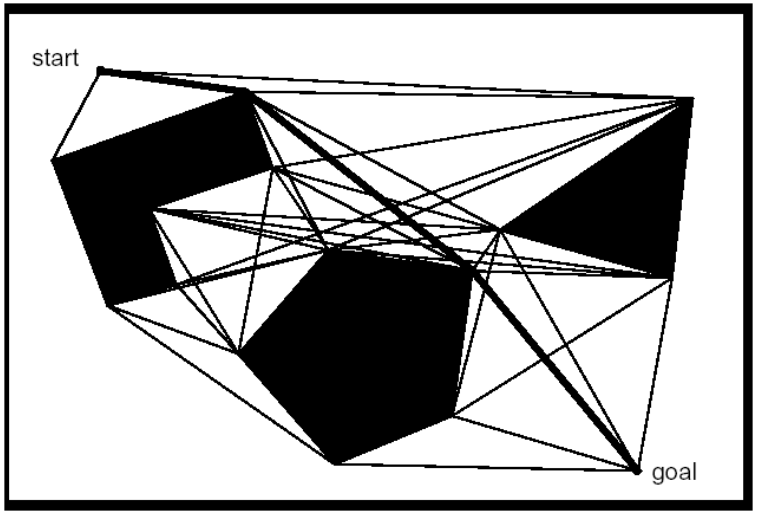

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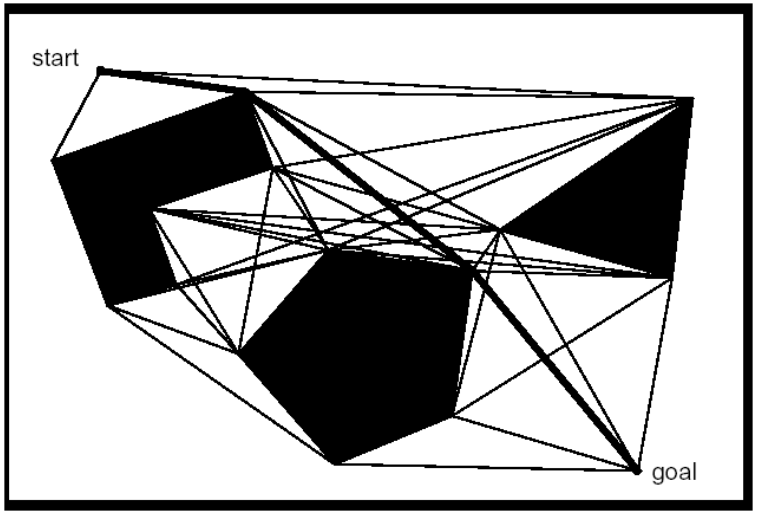

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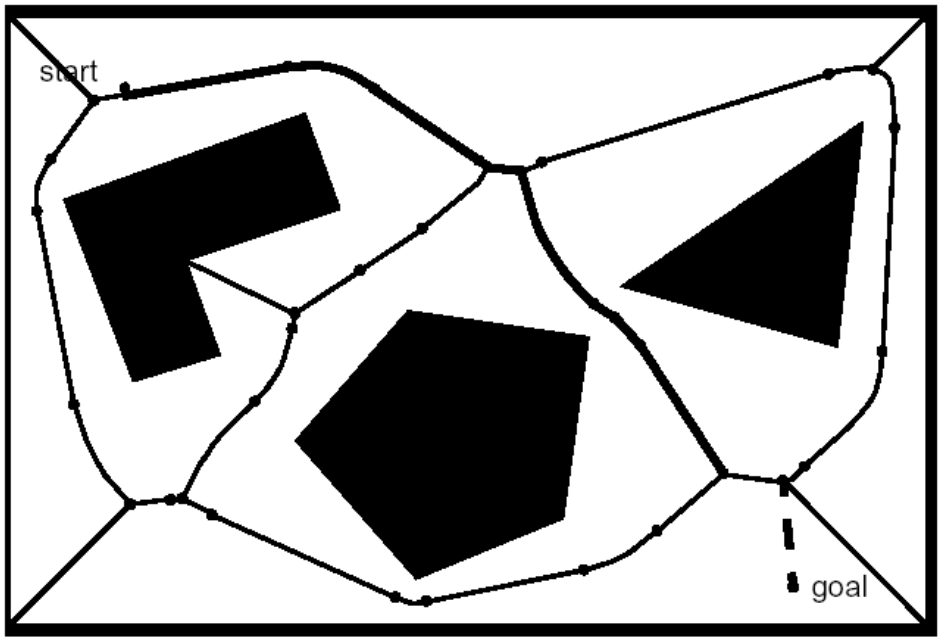

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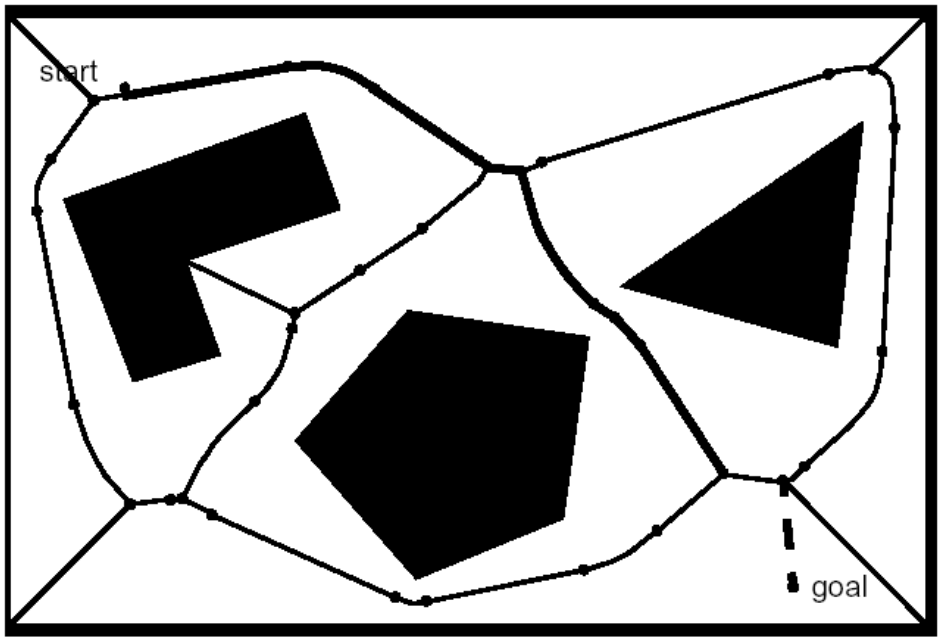

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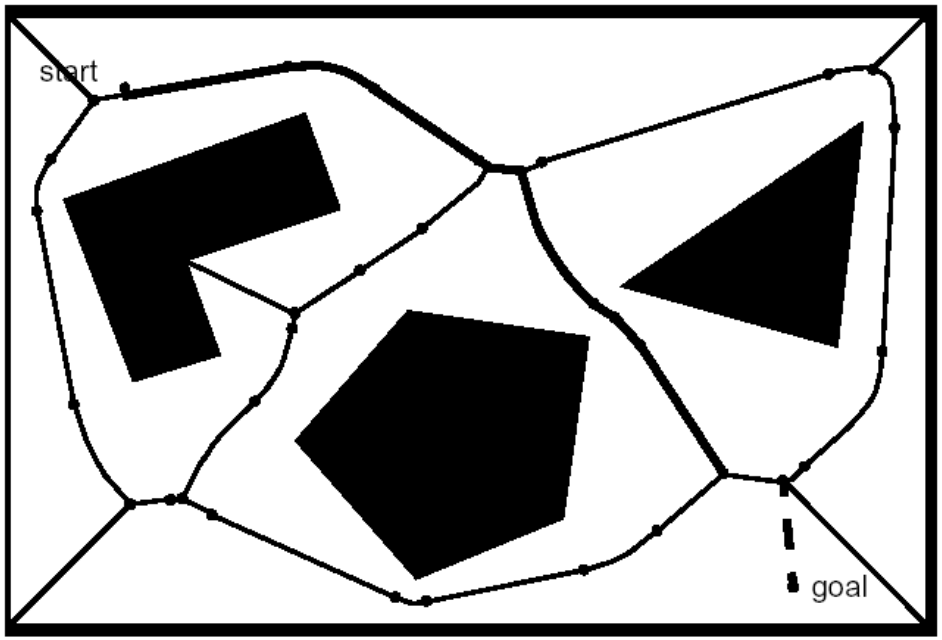

Paths are safer, but longer, than those of visibility graphs. A robot not on the GVD can easily join it by moving away from the nearest obstacle until the GVD is reached.

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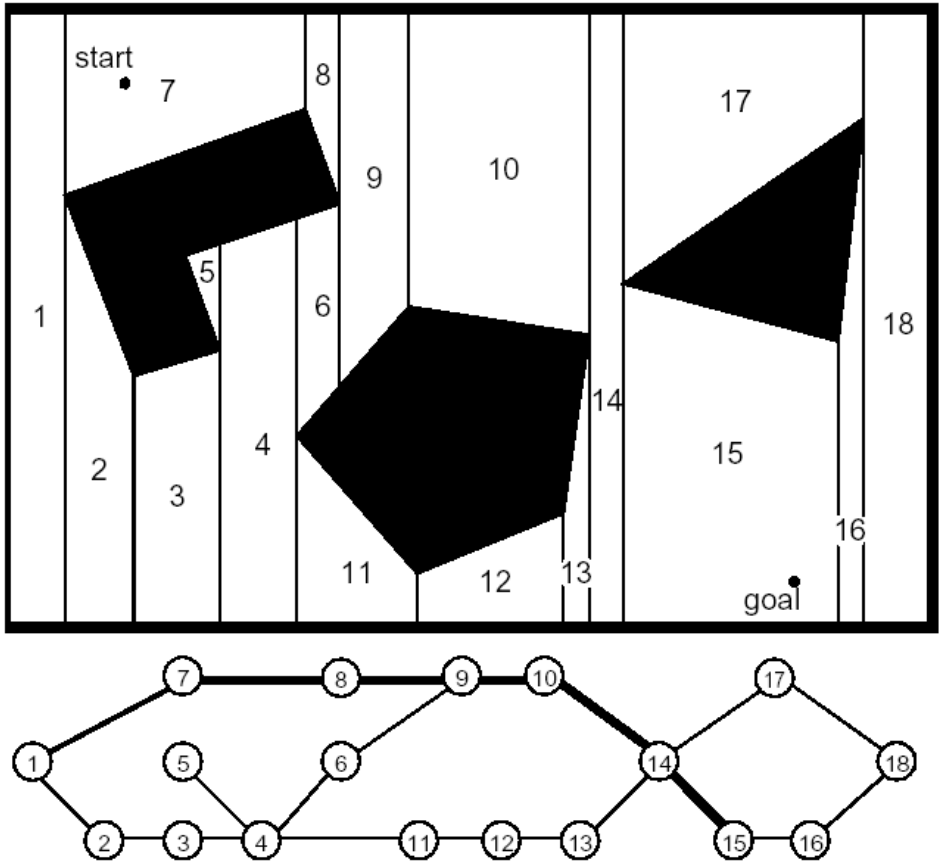

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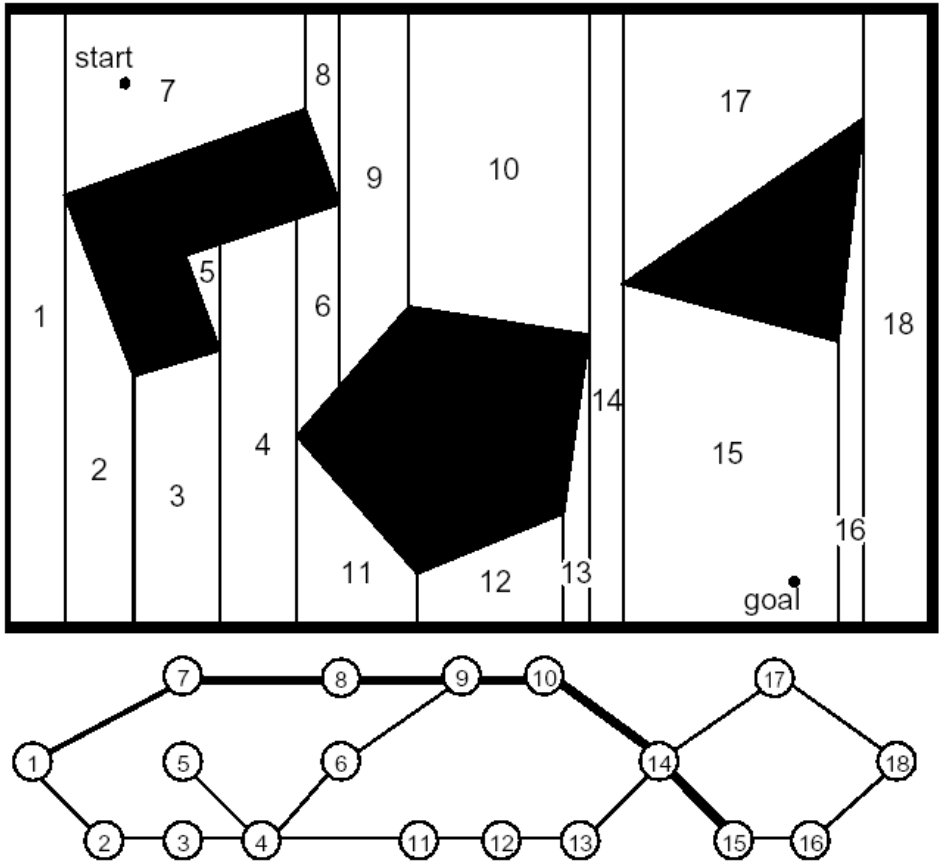

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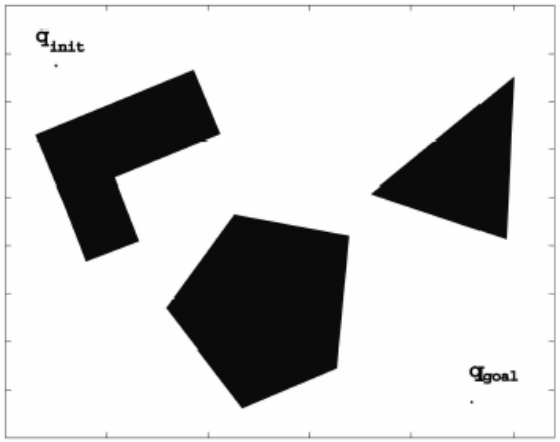
This technique is more than just a path planning device. The resulting vectors provide a low level control law for guiding the robot.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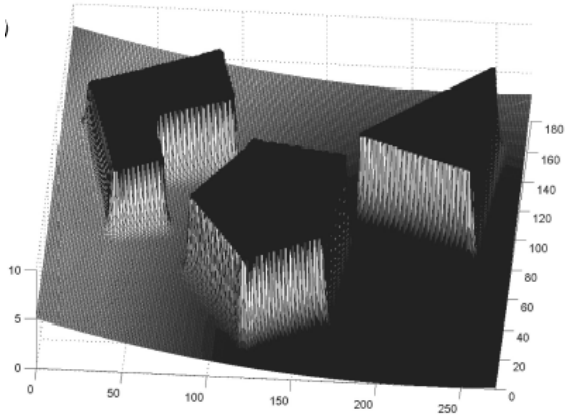
q_{goal}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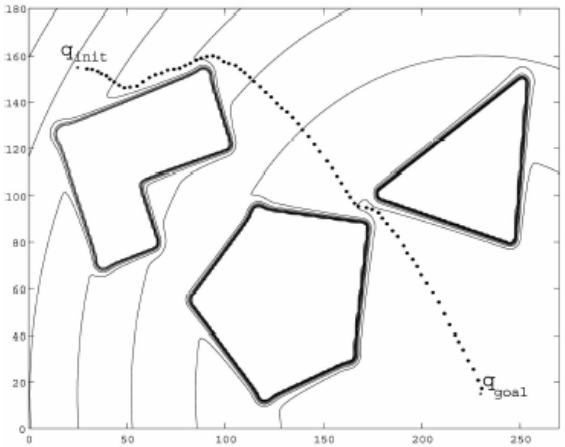
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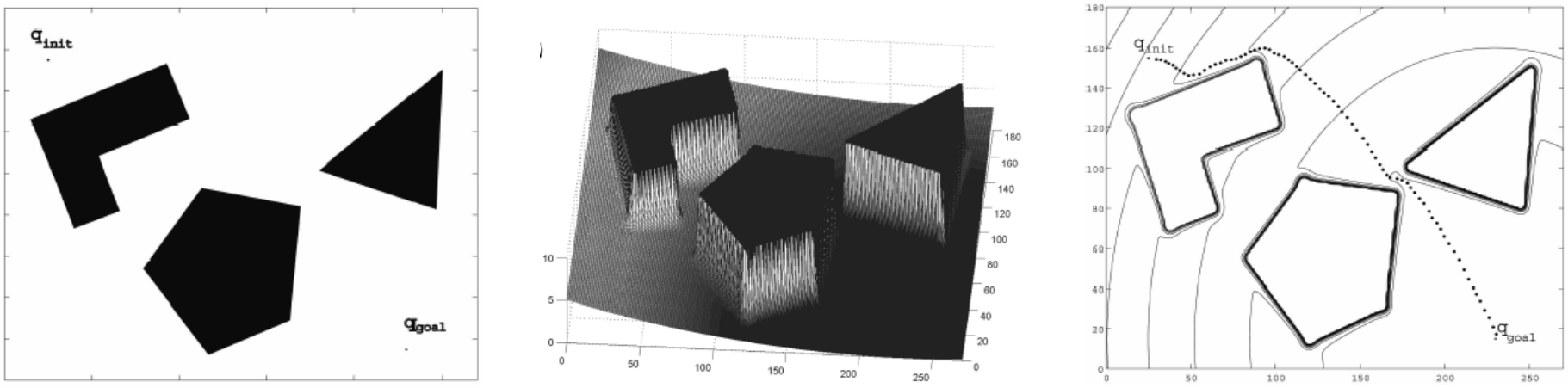

Left: Obstacle map

Centre: Potential field

Right: Gradient contour and path followed