Perception, Part 2
Range-Based Feature Extraction

Computer Science 4766/6912

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Memorial University of Newfoundland

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1. Feature Extraction
   - Line Extraction
   - Other Range-Based Features

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- Features are very useful for localization and mapping

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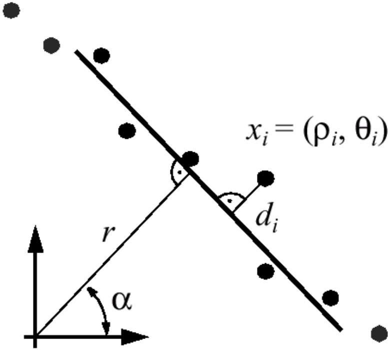
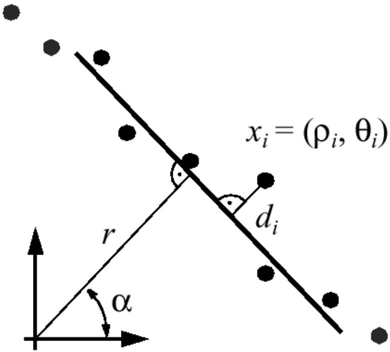
  - We apply *least squares optimization* to find a line of best fit
  - In fact, we apply *weighted least squares* using the uncertainty of data points to weight their impact on the solution


$x_i = (\rho_i, \theta_i)$
d_i
r
α



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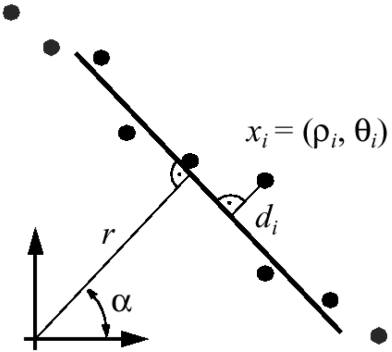

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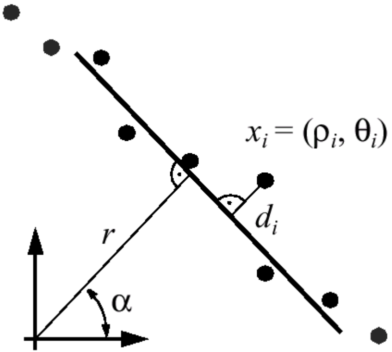

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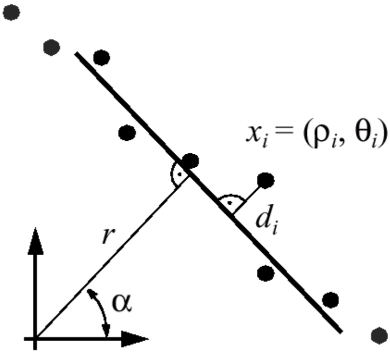

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We need to formulate the orthogonal distance d_i from a data point (ρ_i, θ_i) to a line with parameters r and α...

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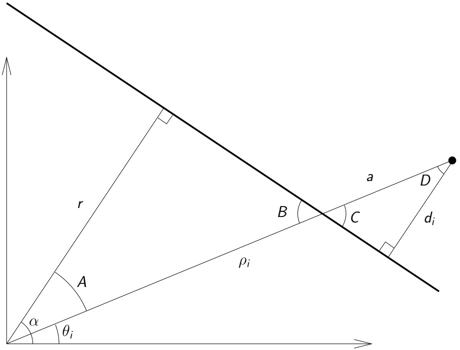

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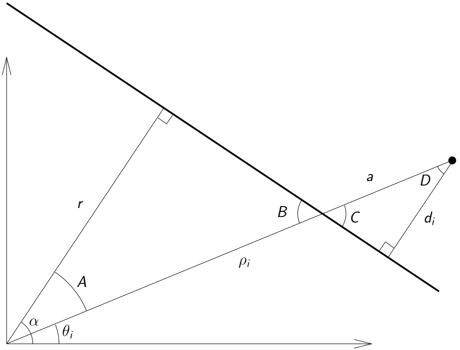

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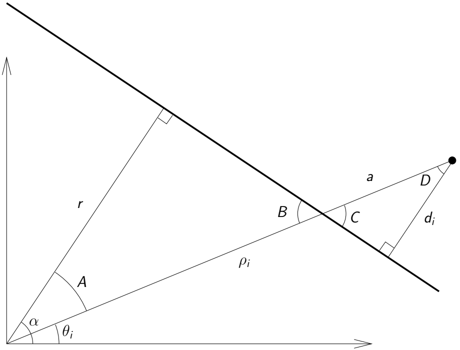

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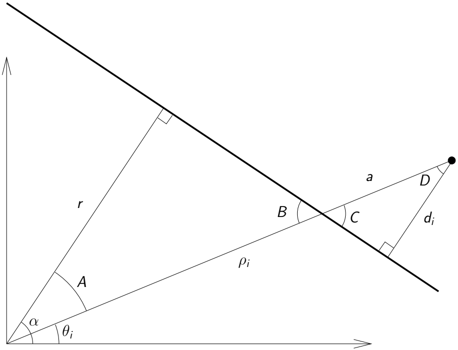

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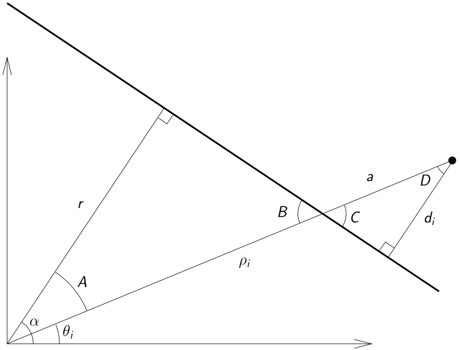

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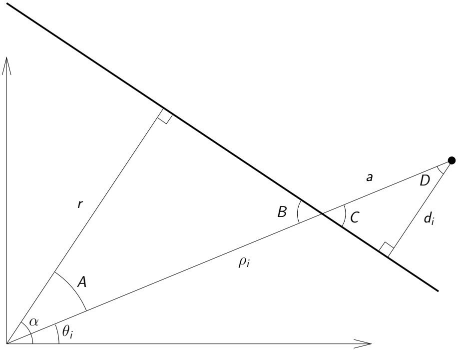

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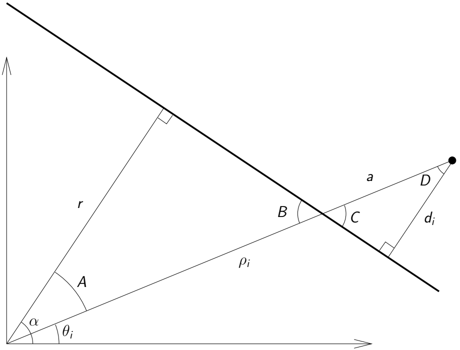

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$$\begin{aligned} \cos(\alpha - \theta_i) &= \frac{d_i}{a} \\ d_i &= \rho_i \cos(\theta_i - \alpha) - r \end{aligned}$$

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$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

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$$= -2\sum w_i \rho_i (\cos\theta_i \cos\alpha + \sin\theta_i \sin\alpha) + 2\sum w_i r$$

$$= -2\sum w_i \rho_i \cos(\theta_i - \alpha) + 2r\sum w_i$$

$$2r\sum w_i = 2\sum w_i \rho_i \cos(\theta_i - \alpha)$$

$$r = \frac{\sum w_i \rho_i \cos(\theta_i - \alpha)}{\sum w_i}$$

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You should use `atan2` to return a result in $[-\pi, \pi]$

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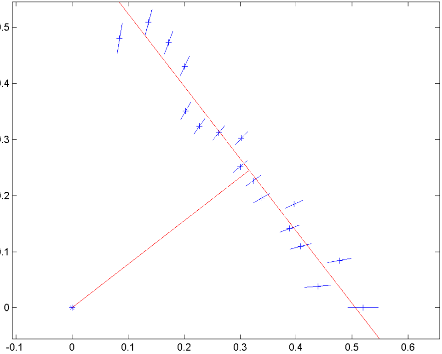

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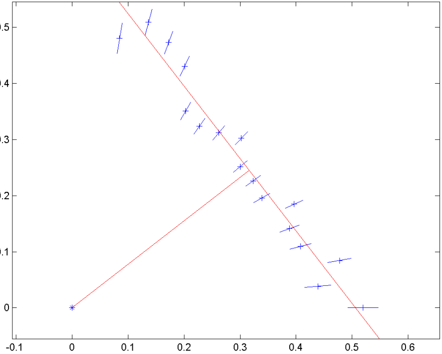

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We introduce a parameter called `minPointsPerLine` to prevent fitting lines to small sets of points to reduce the impact of noise

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More details are required for steps 3 and 6...

Step 3:

We introduce a parameter called `minPointsPerLine` to prevent fitting lines to small sets of points to reduce the impact of noise. Therefore, the worst fit point should not be one of the first or last `minPointsPerLine`/2 points of the list.

Further details...

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(The method described above is similar, but slightly simplified from that described in [Nguyen et al., 2005])

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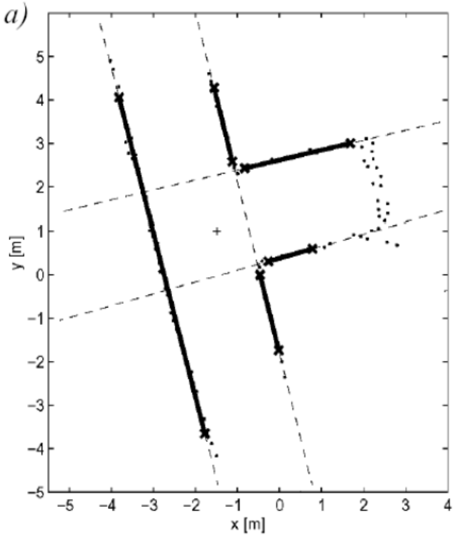

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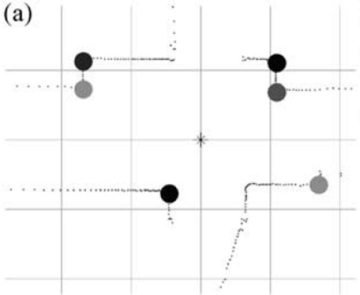

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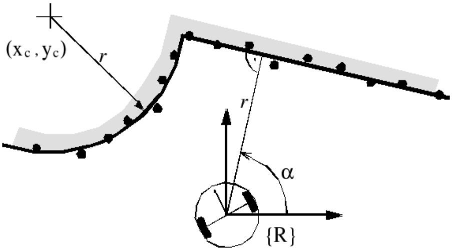

Model for *circles:*

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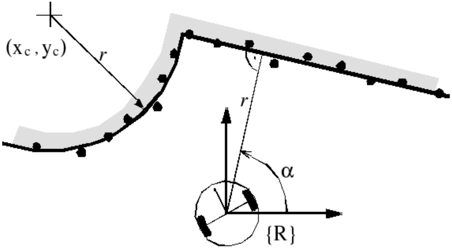

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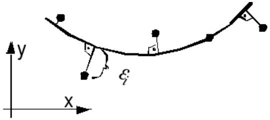

$$(x_i + x_c)^2 + (y_i + y_c)^2 - r^2 = \varepsilon_i$$

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