Unit 4: Localization Part 4
Monte Carlo Localization

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

June 11, 2018

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- For a derivation of the algorithm see [Thrun et al., 2005]

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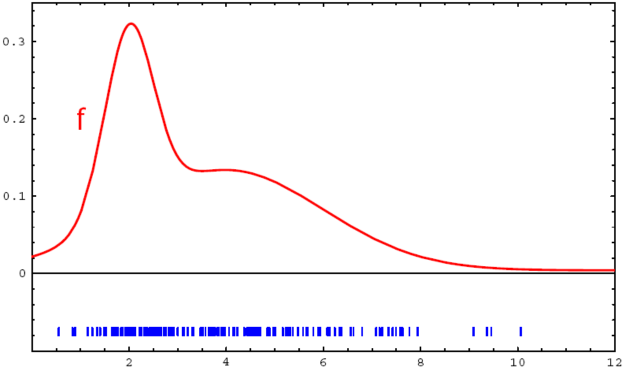

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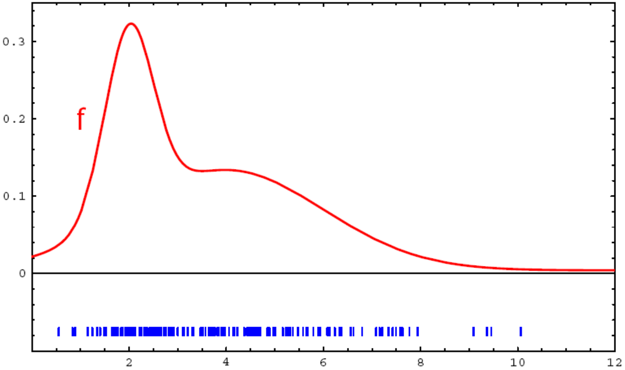

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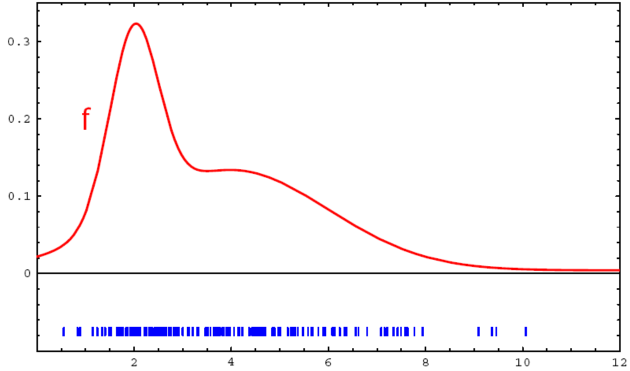

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A particle filter is just the application of Bayes filter to estimate a probability distribution, where that distribution is represented by a set of samples

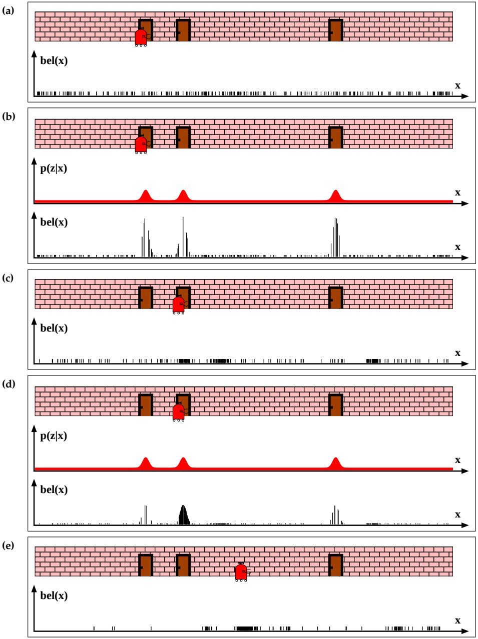

Figure 8.11 Monte Carlo Localization, a particle filter applied to mobile robot localization.

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where M is the number of particles

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The set of particles $\overline{X}_t$ represents $\overline{bel}(x_t)$; But how does this ‘sample’ step work?

Consider again the odometry motion model:

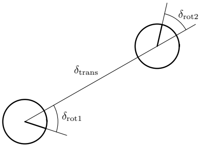

Figure 5.7 Odometry model: The robot motion in the time interval $(t-1, t]$ is approximated by a rotation δ_{rot1}, followed by a translation δ_{trans} and a second rotation δ_{rot2}. The turns and translations are noisy.

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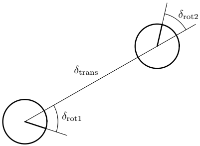

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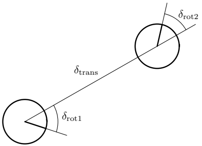

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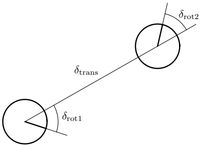

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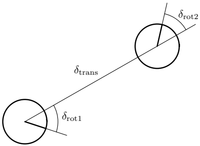

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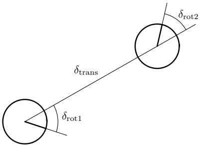

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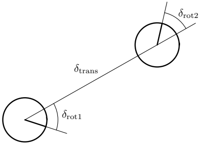

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Method: We obtain the δ parameters from u_t (as before)

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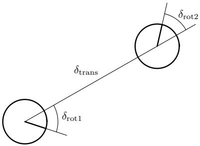

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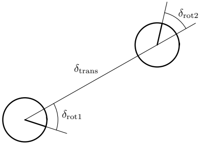

Figure 5.7 Odometry model: The robot motion in the time interval $(t-1, t]$ is approximated by a rotation $\delta_{\rm rot1}$, followed by a translation $\delta_{\rm trans}$ and a second rotation $\delta_{\rm rot2}$. The turns and translations are noisy.

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Method: We obtain the δ parameters from u_t (as before); We then add random noise to these to generate the $\hat{\delta}$ parameters ; Finally, we compute x_t as a movement from x_{t-1} , specified by the $\hat{\delta}$ parameters

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return $x_t = (x', y', \theta')^T$

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$\quad \theta' = \theta + \hat{\delta}_{rot1} + \hat{\delta}_{rot2}$

$\quad$ return $x_t = (x', y', \theta')^T$

where $\mathrm{sample}(\sigma^2)$ draws a random value from a Gaussian distribution with variance σ^2

Shown below are 500 samples obtained from sample_motion_model_odometry using three different sets of *alpha* parameters, Shown below are 500 samples obtained from sample_motion_model_odometry using three different sets of *alpha* parameters,

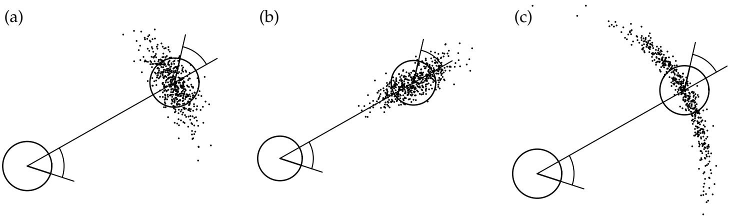

Figure 5.9 Sampling from the odometry motion model, using the same parameters as in Figure 5.8. Each diagram shows 500 samples.

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To obtain $bel(x_t)$ we must take measurements into account; This is accomplished by giving each particle, $x_t^{[m]}$, a weight of $p(z_t|x_t^{[m]})$

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Incorporating this step yields the full particle filter algorithm...

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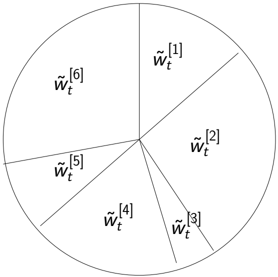

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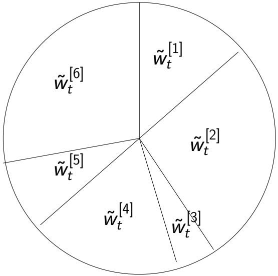

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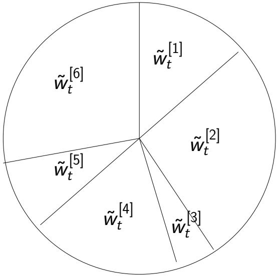

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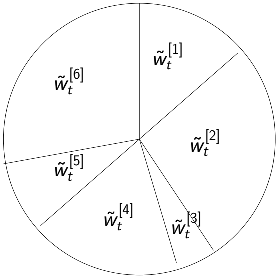

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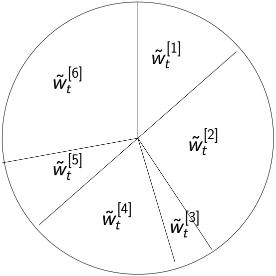

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$$\tilde{w}_t^{[i]} = \frac{w_t^{[i]}}{s}, \qquad \text{where } s = \sum_j w_t^{[j]}$$

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From the particles in the prediction step, $\overline{X}_t$, we select M particles to go into X_t; Often we will get many copies of the same particle carrying over into X_t; Thus, resampling reduces diversity and focusses particles on areas of the state space where $bel(x_t)$ is large

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- Revisit figure 8.11

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- They are (relatively) easy to implement!

References

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