Unit 4: Localization Part 3
Motion and Measurement Models + Grid Localization

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

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1. Odometry Motion Model

2. Measurement Model

3. Grid Localization
We require a general motion model to apply Bayes filter to mobile robot localization.
Odometry Motion Model: $p(x_t | u_t, x_{t-1})$

- We require a general motion model to apply Bayes filter to mobile robot localization.
- The motion model described here (from [Thrun et al., 2005]) is based on odometry.

We employ the difference between the current odometry pose vector $\bar{x}_t$ and the last odometry pose vector $\bar{x}_{t-1}$:

$$\bar{x}_t - \bar{x}_{t-1} = [\bar{x}'_t, \bar{y}'_t, \bar{\theta}'_t]^T$$

Define the control or action as,

$$u_t = [\bar{x}'_t, \bar{x}_t]^T$$

The difference between $\bar{x}_t$ and $\bar{x}_{t-1}$ is a good estimate of the difference between $x_t$ and $x_{t-1}$. 
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- An initial rotation of angle $\delta_{rot1}$
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{\text{rot}1}$
- A translation of length $\delta_{\text{trans}}$

Figure 5.7 Odometry model: The robot motion in the time interval $(t - 1, t]$ is approximated by a rotation $\delta_{\text{rot}1}$, followed by a translation $\delta_{\text{trans}}$ and a second rotation $\delta_{\text{rot}2}$. The turns and translations are noisy.
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{rot1}$
- A translation of length $\delta_{trans}$
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**Figure 5.7** Odometry model: The robot motion in the time interval $(t - 1, t]$ is approximated by a rotation $\delta_{rot1}$, followed by a translation $\delta_{trans}$ and a second rotation $\delta_{rot2}$. The turns and translations are noisy.
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- An initial rotation of angle $\delta_{rot1}$
- A translation of length $\delta_{trans}$
- A final rotation of angle $\delta_{rot2}$

![Diagram showing the odometry model with $\delta_{rot1}$, $\delta_{trans}$, and $\delta_{rot2}$]

Figure 5.7 Odometry model: The robot motion in the time interval $(t - 1, t]$ is approximated by a rotation $\delta_{rot1}$, followed by a translation $\delta_{trans}$ and a second rotation $\delta_{rot2}$. The turns and translations are noisy.

We model the robot’s motion using these three parameters.
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{\text{rot1}}$
- A translation of length $\delta_{\text{trans}}$
- A final rotation of angle $\delta_{\text{rot2}}$

**Figure 5.7** Odometry model: The robot motion in the time interval $(t - 1, t]$ is approximated by a rotation $\delta_{\text{rot1}}$, followed by a translation $\delta_{\text{trans}}$ and a second rotation $\delta_{\text{rot2}}$. The turns and translations are noisy.

We model the robot’s motion using these three parameters; Yet the actual motion may have been quite different (e.g. rotating while translating)
Given $u_t = [ \bar{x}_{t-1}, \bar{x}_t ]^T$ we can derive $\delta_{rot1}$, $\delta_{trans}$, and $\delta_{rot2}$ via simple geometry.

$\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$

$\delta_{trans} = \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2}$

$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$
Given \( u_t = [ \bar{x}_{t-1}, \bar{x}_t ]^T \) we can derive \( \delta_{rot1}, \delta_{trans}, \) and \( \delta_{rot2} \) via simple geometry.

**COVERED ON BOARD**
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\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}
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Given \( u_t = [ \bar{x}_{t-1}, \bar{x}_t ]^T \) we can derive \( \delta_{\text{rot}1}, \delta_{\text{trans}}, \) and \( \delta_{\text{rot}2} \) via simple geometry.

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\[
\begin{align*}
\delta_{\text{rot}1} &= \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta} \\
\delta_{\text{trans}} &= \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2}
\end{align*}
\]
Given \( u_t = [ \tilde{x}_{t-1}, \tilde{x}_t ]^T \) we can derive \( \delta_{rot1}, \delta_{trans}, \) and \( \delta_{rot2} \) via simple geometry.

**COVERED ON BOARD**

\[
\begin{align*}
\delta_{rot1} &= \text{atan2}(\tilde{y}' - \tilde{y}, \tilde{x}' - \tilde{x}) - \bar{\theta} \\
\delta_{trans} &= \sqrt{(\tilde{x} - \tilde{x}')^2 + (\tilde{y} - \tilde{y}')^2} \\
\delta_{rot2} &= \bar{\theta}' - \bar{\theta} - \delta_{rot1}
\end{align*}
\]
Given \( u_t = [\bar{x}_{t-1}, \bar{x}_t]^T \) we can derive \( \delta_{rot1}, \delta_{trans}, \) and \( \delta_{rot2} \) via simple geometry.

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\begin{align*}
\delta_{rot1} &= \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta} \\
\delta_{trans} &= \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2} \\
\delta_{rot2} &= \bar{\theta}' - \bar{\theta} - \delta_{rot1}
\end{align*}
\]
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\delta_{\text{true}}^{\text{rot}1} = \delta_{\text{rot}1} - \epsilon_{\text{rot}1}
\]

\[
\delta_{\text{true}}^{\text{trans}} = \delta_{\text{trans}} - \epsilon_{\text{trans}}
\]

\[
\delta_{\text{true}}^{\text{rot}2} = \delta_{\text{rot}2} - \epsilon_{\text{rot}2}
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We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\delta_{\text{true}}^{\text{rot}1} = \delta_{\text{rot}1} - \epsilon_{\text{rot}1}
\]

\[
\delta_{\text{true}}^{\text{trans}} = \sqrt{(x - x')^2 + (y - y')^2}
\]

\[
\delta_{\text{true}}^{\text{rot}2} = \theta' - \theta - \hat{\delta}_{\text{rot}1}
\]
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{true\ rot}}^{\text{rot1}} &= \delta_{\text{rot1}} - \epsilon_{\text{rot1}} \\
\delta_{\text{true\ trans}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}}
\end{align*}
\]
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{true rot} 1} &= \delta_{\text{rot} 1} - \epsilon_{\text{rot} 1} \\
\delta_{\text{true trans}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\delta_{\text{true rot} 2} &= \delta_{\text{rot} 2} - \epsilon_{\text{rot} 2}
\end{align*}
\]
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{true rot1}} &= \delta_{\text{rot1}} - \epsilon_{\text{rot1}} \\
\delta_{\text{true trans}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\delta_{\text{true rot2}} &= \delta_{\text{rot2}} - \epsilon_{\text{rot2}}
\end{align*}
\]

where the three \( \epsilon \)'s are Normal random variables assumed to have zero mean.
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{rot}1}^{\text{true}} & = \delta_{\text{rot}1} - \epsilon_{\text{rot}1} \\
\delta_{\text{trans}}^{\text{true}} & = \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\delta_{\text{rot}2}^{\text{true}} & = \delta_{\text{rot}2} - \epsilon_{\text{rot}2}
\end{align*}
\]

where the three $\epsilon$'s are Normal random variables assumed to have zero mean.

We don’t know these true parameters.
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\delta_{\text{rot2}}^{\text{true}} &= \delta_{\text{rot2}} - \epsilon_{\text{rot2}}
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We don’t know these true parameters. But given a pair of poses \( x_t \) and \( x_{t-1} \) we can compute the parameters we would expect
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

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\[
x_t = [x', y', \theta']^T \quad x_{t-1} = [x, y, \theta]^T
\]
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta^\text{true}_{\text{rot1}} &= \delta_{\text{rot1}} - \epsilon_{\text{rot1}} \\
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x_t = [x', y', \theta']^T \quad x_{t-1} = [x, y, \theta]^T
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\[
\hat{\delta}_{\text{rot1}} = \text{atan2}(y' - y, x' - x) - \theta
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We assume that these measured values are corrupted by independent noise, such that the true values are given as follows,

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\begin{align*}
\delta_{\text{rot}1}^{\text{true}} &= \delta_{\text{rot}1} - \epsilon_{\text{rot}1} \\
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\delta_{\text{rot}2}^{\text{true}} &= \delta_{\text{rot}2} - \epsilon_{\text{rot}2}
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\]

\[
\begin{align*}
\hat{\delta}_{\text{rot}1} &= \text{atan2}(y' - y, x' - x) - \theta \\
\hat{\delta}_{\text{trans}} &= \sqrt{(x - x')^2 + (y - y')^2}
\end{align*}
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We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

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\hat{\delta}_{\text{rot}2} = \theta' - \theta - \hat{\delta}_{\text{rot}1}
\]
In order to work out the probability of a given movement, we need to know the variance of the noise processes, $\epsilon_{rot1}$, $\epsilon_{trans}$, and $\epsilon_{rot2}$. We utilize the equations below to model the relationship between the amount of motion and the amount of uncertainty:

\[
\sigma^2_{rot1} = \alpha_1 \delta^2_{rot1} + \alpha_2 \delta^2_{trans}
\]

\[
\sigma^2_{trans} = \alpha_3 \delta^2_{trans} + \alpha_4 (\delta^2_{rot1} + \delta^2_{rot2})
\]

\[
\sigma^2_{rot2} = \alpha_1 \delta^2_{rot2} + \alpha_2 \delta^2_{trans}
\]

where the $\alpha$ parameters specify the noise characteristics of a particular robot (to be determined experimentally).

Note: This model differs from the book which relates the $\sigma$ quantities to the $\hat{\delta}$ quantities.
In order to work out the probability of a given movement, we need to know the variance of the noise processes, $\epsilon_{\text{rot}1}$, $\epsilon_{\text{trans}}$, and $\epsilon_{\text{rot}2}$. We assume that the amount of noise is proportional to the amount of movement, as measured by odometry.
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\sigma^2_{\text{rot}2} &= \alpha_1 \delta^2_{\text{rot}2} + \alpha_2 \delta^2_{\text{trans}}
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The odometry motion model computes \( p(x_t | u_t, x_{t-1}) \) as follows:
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- Compute $\delta_{rot1}, \delta_{trans},$ and $\delta_{rot2}$ from odometry
  - $\bar{x}_t$ is computed by integrating wheel motion; $\bar{x}_{t-1}$ is simply the same result from the last time step
- Compute $\hat{\delta}_{rot1}, \hat{\delta}_{trans},$ and $\hat{\delta}_{rot2}$ for a specific pair of $x_t$ and $x_{t-1}$
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  - \( \bar{x}_t \) is computed by integrating wheel motion; \( \bar{x}_{t-1} \) is simply the same result from the last time step
- Compute \( \hat{\delta}_{rot1}, \hat{\delta}_{trans}, \) and \( \hat{\delta}_{rot2} \) for a specific pair of \( x_t \) and \( x_{t-1} \)
- Assuming for the moment that the “hat” parameters are true, determine the probability of obtaining \( \delta_{rot1}, \delta_{trans}, \) and \( \delta_{rot2} \)
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- Compute $\hat{\delta}_{rot1}, \hat{\delta}_{trans}$, and $\hat{\delta}_{rot2}$ for a specific pair of $x_t$ and $x_{t-1}$
- Assuming for the moment that the “hat” parameters are true, determine the probability of obtaining $\delta_{rot1}, \delta_{trans}$, and $\delta_{rot2}$
This probability is computed as follows:
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\[ p_1 = \text{prob}(\delta_{\text{rot}1} - \hat{\delta}_{\text{rot}1}, \sigma^2_{\text{rot}1}) \]
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\[ p_1 = \text{prob}(\delta_{\text{rot}1} - \hat{\delta}_{\text{rot}1}, \sigma^2_{\text{rot}1}) \]
\[ p_2 = \text{prob}(\delta_{\text{trans}} - \hat{\delta}_{\text{trans}}, \sigma^2_{\text{trans}}) \]
This probability is computed as follows:

\[
p_1 = \text{prob}(\delta_{\text{rot}1} - \hat{\delta}_{\text{rot}1}, \sigma^2_{\text{rot}1})
\]

\[
p_2 = \text{prob}(\delta_{\text{trans}} - \hat{\delta}_{\text{trans}}, \sigma^2_{\text{trans}})
\]

\[
p_3 = \text{prob}(\delta_{\text{rot}2} - \hat{\delta}_{\text{rot}2}, \sigma^2_{\text{rot}2})
\]
This probability is computed as follows:

\[ p_1 = \text{prob}(\delta_{rot1} - \hat{\delta}_{rot1}, \sigma_{rot1}^2) \]
\[ p_2 = \text{prob}(\delta_{trans} - \hat{\delta}_{trans}, \sigma_{trans}^2) \]
\[ p_3 = \text{prob}(\delta_{rot2} - \hat{\delta}_{rot2}, \sigma_{rot2}^2) \]

\[ p(x_t | u_t, x_{t-1}) = p_1 \cdot p_2 \cdot p_3 \]
This probability is computed as follows:

\[
p_1 = \text{prob}(\delta_{\text{rot}1} - \hat{\delta}_{\text{rot}1}, \sigma_{\text{rot}1}^2)
\]

\[
p_2 = \text{prob}(\delta_{\text{trans}} - \hat{\delta}_{\text{trans}}, \sigma_{\text{trans}}^2)
\]

\[
p_3 = \text{prob}(\delta_{\text{rot}2} - \hat{\delta}_{\text{rot}2}, \sigma_{\text{rot}2}^2)
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where \(\text{prob}(v, \sigma^2)\) returns the probability density for value \(v\) of a zero-mean Normal distribution with variance \(\sigma^2\)
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where \(\text{prob}(v, \sigma^2)\) returns the probability density for value \(v\) of a zero-mean Normal distribution with variance \(\sigma^2\). The product of probabilities \(p_1 \cdot p_2 \cdot p_3\) is the joint probability under the assumption that the three noise processes are independent.
The shape of \( p(x_t|u_t, x_{t-1}) \) depends on the \( \alpha_i \) parameters, which should be chosen to match (or exceed) the robot’s actuator and odometry errors.
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Figure 5.8  The odometry motion model, for different noise parameter settings.
Measurement Model: $p(z_t|x_t)$

We must determine the probability of observing $z_t$ given a pose $x_t$. The measurement model depends heavily on the map, $m$. 

Figure 6.1 (a) Typical ultrasound scan of a robot in its environment. (b) A misreading in ultrasonic sensing. This effect occurs when firing a sonar signal towards a reflective surface at an angle $\alpha$ that exceeds half the opening angle of the sensor.
Measurement Model: \[ p(z_t|x_t) \]

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![Diagram](image)

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$$p(z_t|x_t) = \prod_{k=1}^{K} p(z_k^t|x_t)$$
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We **cast a ray** from position $\mathbf{x}_t$ in the map in the direction that our sensor is facing
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Thus, if we know $z_t^{k*}$ and $\sigma_{hit}$ we can calculate $p_{hit}(z_t^k|x_t)$. But how do we determine $z_t^{k*}$?

We cast a ray from position $x_t$ in the map in the direction that our sensor is facing. When this ray hits its first obstacle we set $z_t^{k*}$ to the distance it has travelled.
We may wish to restrict the distribution to have zero probability after the sensor’s maximum range $z_{max}$, as we know the sensor will not produce a larger value.
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(a) Gaussian distribution $p_{\text{hit}}$

$$p(z_t^k \mid x_t, m)$$

$z_t^k$ $z_{\text{max}}$
2. Unexpected objects

The map may omit many objects (e.g. people). The presence of such an object will tend to reduce the reported range. The probability of an object interposing itself between the robot and a mapped part of the environment decreases with range. This probability can be modelled as an exponential distribution, truncated at the true range.
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(b) Exponential distribution $p_{\text{short}}$

$$p(z_t^k \mid x_t, m)$$
3. Missed objects

The sensor may miss an object altogether. Or the emitted beam may experience specular reflection and never return to the sensor. This possibility is modelled by a “spike” in probability at the maximum range of the sensor $z_{\text{max}}$. 

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(c) Uniform distribution $p_{max}$

$$p(z_t^k \mid x_t, m)$$

$z_{t*}$

$z_{max}$
4. Unexplainable measurements

Sometimes sensors produce inexplicable measurements, for no apparent reason. We "model" such occurrences by a uniform distribution.
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(d) Uniform distribution $p_{\text{rand}}$

$$p(z_t^k \mid x_t, m)$$
The following shows all four of these densities individually:

\[ p(z_k^t | x_t, m) \]

- **Gaussian distribution**:\( p_{hit} \)
- **Exponential distribution**:\( p_{short} \)
- **Uniform distribution**:\( p_{max} \)
- **Uniform distribution**:\( p_{rand} \)

**Figure 6.3** Components of the range finder sensor model. In each diagram the horizontal axis corresponds to the measurement \( z_k^t \), the vertical to the likelihood.
The following shows all four of these densities individually:

(a) Gaussian distribution $p_{hit}$

(b) Exponential distribution $p_{short}$

(c) Uniform distribution $p_{max}$

(d) Uniform distribution $p_{rand}$

**Figure 6.3** Components of the range finder sensor model. In each diagram the horizontal axis corresponds to the measurement $z_t^k$, the vertical to the likelihood.
A weighted combination of the four densities gives \( p(z^k_t|x_t) \):
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\[ p(z_t^k | x_t, m) \]

**Figure 6.4** “Pseudo-density” of a typical mixture distribution $p(z_t^k | x_t, m)$. 
Figure 6.7  Probabilistic model of perception: (a) Laser range scan, projected into a previously acquired map $m$. (b) The likelihood $p(z_t \mid x_t, m)$, evaluated for all positions $x_t$ and projected into the map (shown in gray). The darker a position, the larger $p(z_t \mid x_t, m)$. 
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(a) Laser scan and part of the map

(b) Likelihood for different positions
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- Fine-grained approaches yield good results but with a high computational cost
- Has the ability to track multiple hypotheses
- The measurement model is usually defined on raw sensor values → feature extraction not required
Figure 8.1 Grid localization using a fine-grained metric decomposition. Each picture depicts the position of the robot in the hallway along with its belief $bel(x_t)$, represented by a histogram over a grid.
For robots operating in the plane we require a 3-D probability cube to represent our belief in the robot’s pose $x_t = [x, y, \theta]^T$. 

Figure 8.2 Example of a fixed-resolution grid over the robot pose variables $x$, $y$, and $\theta$. Each grid cell represents a robot pose in the environment. Different orientations of the robot correspond to different planes in the grid (shown are only three orientations).
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![Diagram](image)

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![Occupancy grid map](image)

**Figure 8.8** Occupancy grid map of the 1994 AAAI mobile robot competition arena.
• Typical resolution of probability cube: 15 cm in $x$ and $y$, $5^\circ$ in $\theta$
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  • Application of the measurement model iterates over all $x_t$, and for each $x_t$ one must iterate over all $k$ sensor values. For range sensors, each scan point requires a ray casting operation.
There are a number of ways of speeding up grid localization:

- Reduce frequency of updates.
- Decrease grid resolution: Problems: decreases accuracy, may require exaggerated noise in motion model.
- Selective updating: Apply updates in local neighbourhood: When updating a cell, restrict the space of possible prior poses to those centred around that cell.
Computational considerations

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- Reduce the number of sensors used.
An example of grid localization

It is important to note that the probabilities in the grid have been projected from 3-D to 2-D. The distribution of belief over \( \theta \) is very important, but it is not shown below.

Figure 8.9 (a) Data set (odometry and sonar range scans) collected in the environment shown in Figure 8.8. This data set is sufficient for global localization using the grid localization. The beliefs at the points marked "A," "B," and "C" are shown in (b), (c), and (d).
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