Unit 4: Localization Part 3
Motion and Measurement Models + Grid Localization

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

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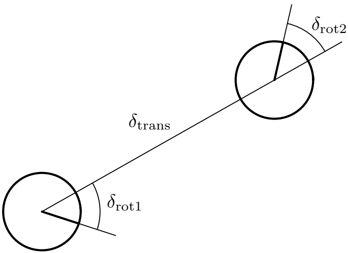

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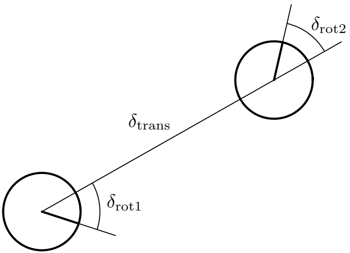

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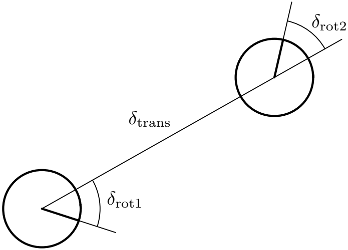

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We model the robot's motion using these three parameters; Yet the actual motion may have been quite different (e.g. rotating while translating)

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COVERED ON BOARD

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Note: This model differs from the book which relates the σ quantities to the $\hat{\delta}$ quantities.

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where $\mathrm{prob}(v, \sigma^2)$ returns the probability density for value v of a zero-mean Normal distribution with variance σ^2. The product of probabilities $p_1 \cdot p_2 \cdot p_3$ is the joint probability under the assumption that the three noise processes are independent.

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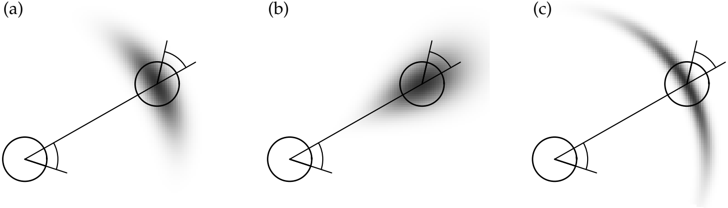

Figure 5.8 The odometry motion model, for different noise parameter settings.

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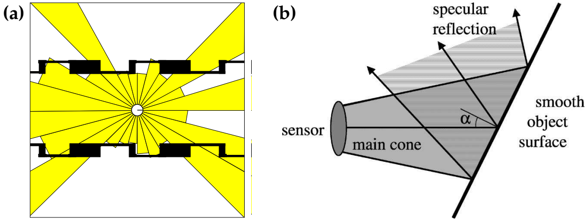

Figure 6.1 (a) Typical ultrasound scan of a robot in its environment. (b) A misreading in ultrasonic sensing. This effect occurs when firing a sonar signal towards a reflective surface at an angle α that exceeds half the opening angle of the sensor.

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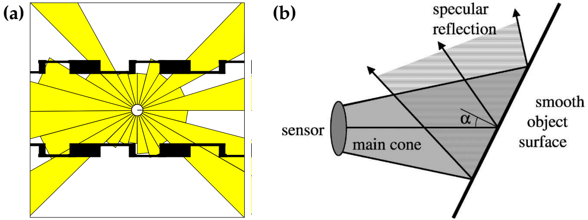

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The measurement model depends heavily on the map, m

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We **cast a ray** from position x_t in the map in the direction that our sensor is facing. When this ray hits its first obstacle we set z_t^{k*} to the distance it has travelled.

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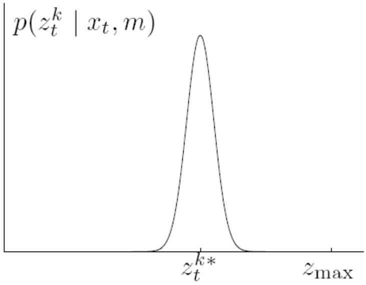
(a) Gaussian distribution p_{hit}



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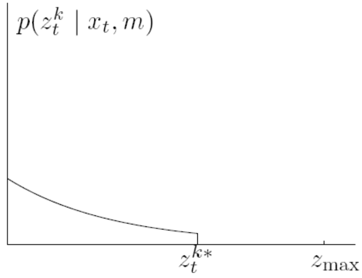
(b) Exponential distribution p_{short}



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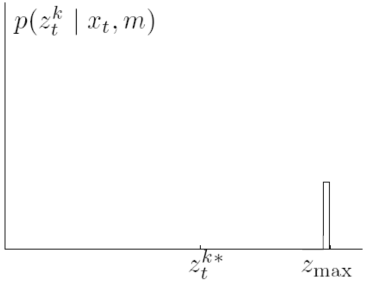
(c) Uniform distribution $p_{\max}$



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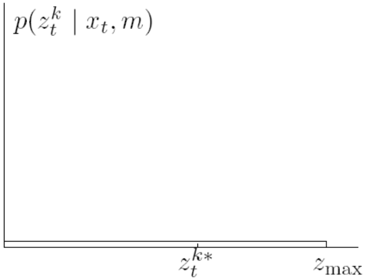
(d) Uniform distribution p_{rand}



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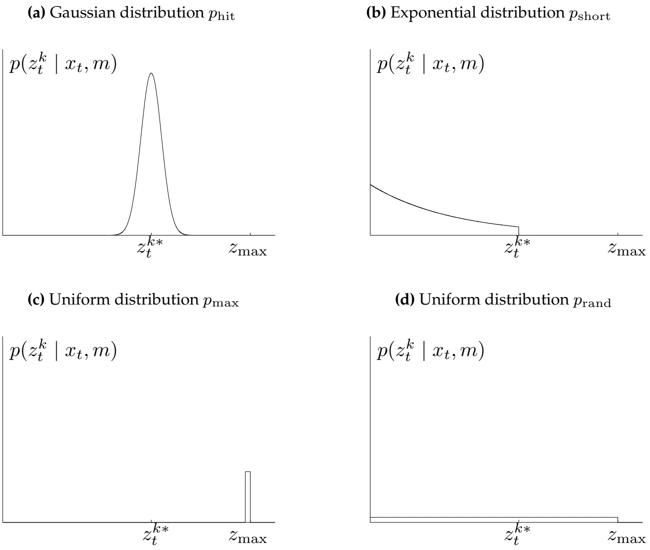

Figure 6.3 Components of the range finder sensor model. In each diagram the horizontal axis corresponds to the measurement z_t^k, the vertical to the likelihood.

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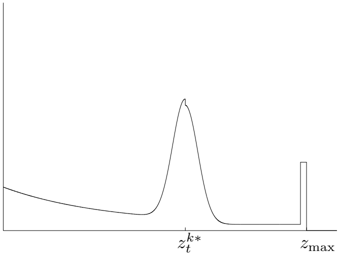
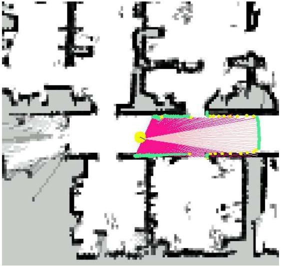


Figure 6.4 "Pseudo-density" of a typical mixture distribution $p(z_t^k \mid x_t, m)$.

(a) Laser scan and part of the map



(b) Likelihood for different positions

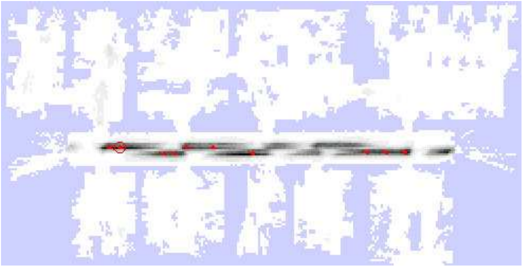


Figure 6.7 Probabilistic model of perception: (a) Laser range scan, projected into a previously acquired map m. (b) The likelihood $p(z_t \mid x_t, m)$, evaluated for all positions x_t and projected into the map (shown in gray). The darker a position, the larger $p(z_t \mid x_t, m)$.

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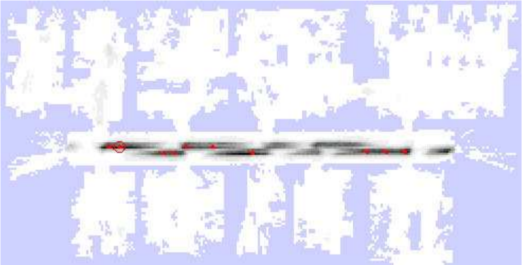

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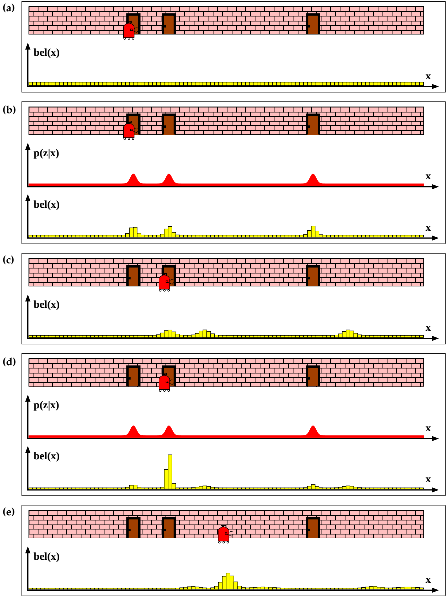

Figure 8.1 Grid localization using a fine-grained metric decomposition. Each picture depicts the position of the robot in the hallway along with its belief $bel(x_t)$, represented by a histogram over a grid.

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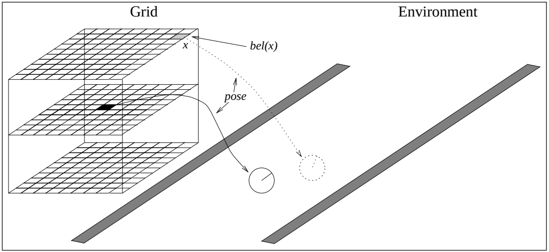

Figure 8.2 Example of a fixed-resolution grid over the robot pose variables x, y, and θ. Each grid cell represents a robot pose in the environment. Different orientations of the robot correspond to different planes in the grid (shown are only three orientations).

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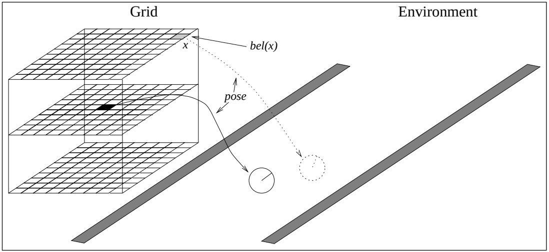

Figure 8.2 Example of a fixed-resolution grid over the robot pose variables x, y, and θ. Each grid cell represents a robot pose in the environment. Different orientations of the robot correspond to different planes in the grid (shown are only three orientations).

Each grid cell contains the probability that the robot has the corresponding pose

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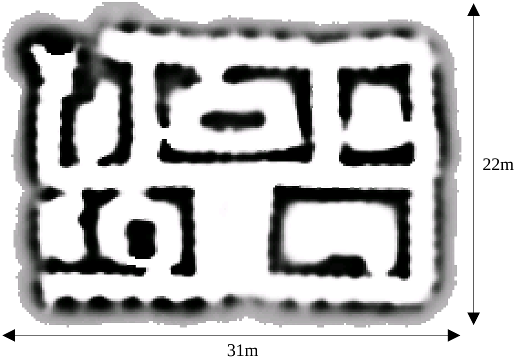

Figure 8.8 Occupancy grid map of the 1994 AAAI mobile robot competition arena.

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  - Application of the measurement model iterates over all x_t, and for each x_t one must iterate over all k sensor values. For range sensors, each scan point requires a ray casting operation.

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  - Problems: decreases accuracy, may require exaggerated noise in motion model.
- Selective updating.
  - Apply updates in local neighbourhood: When updating a cell, restrict the space of possible prior poses to those centred around that cell

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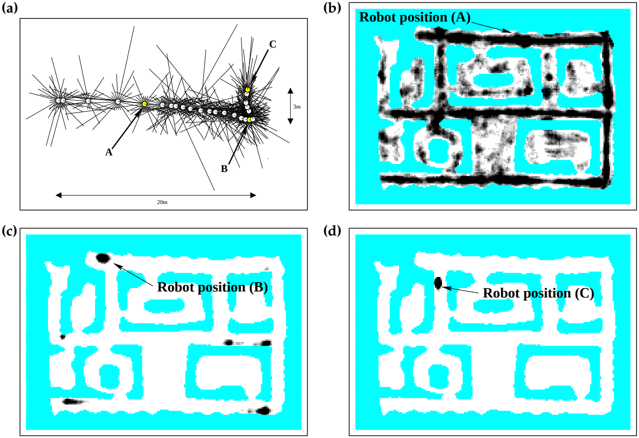

Figure 8.9 (a) Data set (odometry and sonar range scans) collected in the environment shown in Figure 8.8. This data set is sufficient for global localization using the grid localization. The beliefs at the points marked "A," "B" and "C" are shown in (b), (c), and (d).

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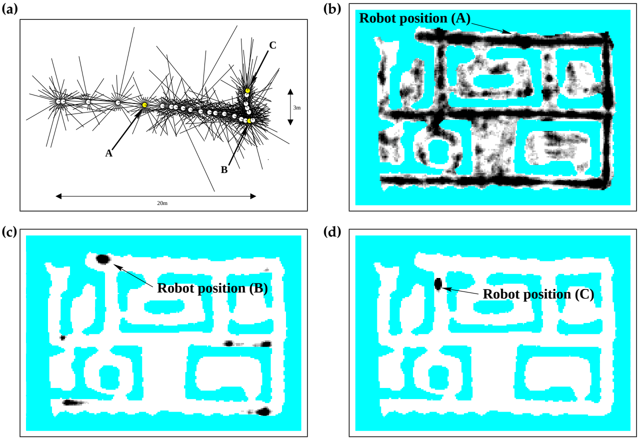

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References

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