Unit 4: Localization Part 3
Motion and Measurement Models + Grid Localization

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

June 11, 2018
1. Odometry Motion Model

2. Measurement Model

3. Grid Localization
Odometry Motion Model: \( p(x_t | u_t, x_{t-1}) \)

- We require a general motion model to apply Bayes filter to mobile robot localization.
Odometry Motion Model: $p(x_t \mid u_t, x_{t-1})$

- We require a general motion model to apply Bayes filter to mobile robot localization.
- The motion model described here (from [Thrun et al., 2005]) is based on odometry.
Odometry Motion Model: \( p(x_t|u_t, x_{t-1}) \)

- We require a general motion model to apply Bayes filter to mobile robot localization
- The motion model described here (from [Thrun et al., 2005]) is based on odometry
- Odometry gives a direct estimate of position \( \rightarrow \) unfortunately this estimate exhibits cumulative error
We require a general motion model to apply Bayes filter to mobile robot localization.

The motion model described here (from [Thrun et al., 2005]) is based on odometry. Odometry gives a direct estimate of position → unfortunately this estimate exhibits cumulative error.

We employ the difference between the current odometry pose vector $\bar{x}_t$ and the last odometry pose vector $\bar{x}_{t-1}$.
We require a general motion model to apply Bayes filter to mobile robot localization.

The motion model described here (from [Thrun et al., 2005]) is based on odometry. Odometry gives a direct estimate of position → unfortunately this estimate exhibits cumulative error.

We employ the difference between the current odometry pose vector $\bar{x}_t$ and the last odometry pose vector $\bar{x}_{t-1}$.

**Odometry Motion Model:** $p(x_t | u_t, x_{t-1})$
We require a general motion model to apply Bayes filter to mobile robot localization.

The motion model described here (from [Thrun et al., 2005]) is based on odometry.

Odometry gives a direct estimate of position → unfortunately this estimate exhibits cumulative error.

We employ the difference between the current odometry pose vector $\bar{x}_t$ and the last odometry pose vector $\bar{x}_{t-1}$

$$\bar{x}_t = [\bar{x}', \bar{y}', \bar{\theta}']^T \quad \bar{x}_{t-1} = [\bar{x}, \bar{y}, \bar{\theta}]^T$$
We require a general motion model to apply Bayes filter to mobile robot localization.

The motion model described here (from [Thrun et al., 2005]) is based on odometry.

Odometry gives a direct estimate of position → unfortunately this estimate exhibits cumulative error.

We employ the difference between the current odometry pose vector \( \bar{x}_t \) and the last odometry pose vector \( \bar{x}_{t-1} \)

\[
\bar{x}_t = [\bar{x}', \bar{y}', \bar{\theta}']^T \quad \bar{x}_{t-1} = [\bar{x}, \bar{y}, \bar{\theta}]^T
\]

Define the control or action as,
Odometry Motion Model: $p(x_t | u_t, x_{t-1})$

- We require a general motion model to apply Bayes filter to mobile robot localization
- The motion model described here (from [Thrun et al., 2005]) is based on odometry
- Odometry gives a direct estimate of position → unfortunately this estimate exhibits cumulative error
- We employ the difference between the current odometry pose vector $\bar{x}_t$ and the last odometry pose vector $\bar{x}_{t-1}$
  \[
  \bar{x}_t = [\bar{x}', \bar{y}', \bar{\theta}']^T, \quad \bar{x}_{t-1} = [\bar{x}, \bar{y}, \bar{\theta}]^T
  \]
- Define the *control* or *action* as,
Odometry Motion Model: $p(x_t | u_t, x_{t-1})$

- We require a general motion model to apply Bayes filter to mobile robot localization.
- The motion model described here (from [Thrun et al., 2005]) is based on odometry.
- Odometry gives a direct estimate of position → unfortunately this estimate exhibits cumulative error.
- We employ the difference between the current odometry pose vector $\bar{x}_t$ and the last odometry pose vector $\bar{x}_{t-1}$
  \[
  \bar{x}_t = [\bar{x}', \bar{y}', \bar{\theta}']^T \\
  \bar{x}_{t-1} = [\bar{x}, \bar{y}, \bar{\theta}]^T
  \]

- Define the control or action as,
  \[
  u_t = [\bar{x}_{t-1}, \bar{x}_t]^T
  \]
Odometry Motion Model: \( p(x_t | u_t, x_{t-1}) \)

- We require a general motion model to apply Bayes filter to mobile robot localization.
- The motion model described here (from [Thrun et al., 2005]) is based on odometry.
- Odometry gives a direct estimate of position → unfortunately this estimate exhibits cumulative error.
- We employ the difference between the current odometry pose vector \( \bar{x}_t \) and the last odometry pose vector \( \bar{x}_{t-1} \)

\[
\bar{x}_t = [\bar{x}', \bar{y}', \bar{\theta}']^T \\
\bar{x}_{t-1} = [\bar{x}, \bar{y}, \bar{\theta}]^T
\]

- Define the control or action as,

\[
u_t = [\bar{x}_{t-1}, \bar{x}_t]^T
\]

- The difference between \( \bar{x}_t \) and \( \bar{x}_{t-1} \) is a good estimate of the difference between \( x_t \) and \( x_{t-1} \).
We transform $u_t$ into a sequence of three steps:
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{rot1}$
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{\text{rot}1}$
- A translation of length $\delta_{\text{trans}}$
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{rot1}$
- A translation of length $\delta_{trans}$
- A final rotation of angle $\delta_{rot2}$
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{rot1}$
- A translation of length $\delta_{trans}$
- A final rotation of angle $\delta_{rot2}$
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{\text{rot}1}$
- A translation of length $\delta_{\text{trans}}$
- A final rotation of angle $\delta_{\text{rot}2}$

**Figure 5.7** Odometry model: The robot motion in the time interval $(t - 1, t]$ is approximated by a rotation $\delta_{\text{rot}1}$, followed by a translation $\delta_{\text{trans}}$ and a second rotation $\delta_{\text{rot}2}$. The turns and translations are noisy.
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{rot1}$
- A translation of length $\delta_{trans}$
- A final rotation of angle $\delta_{rot2}$

We model the robot’s motion using these three parameters.
We transform $u_t$ into a sequence of three steps:

- An initial rotation of angle $\delta_{rot1}$
- A translation of length $\delta_{trans}$
- A final rotation of angle $\delta_{rot2}$

![Diagram of robot motion](image)

**Figure 5.7** Odometry model: The robot motion in the time interval $(t - 1, t]$ is approximated by a rotation $\delta_{rot1}$, followed by a translation $\delta_{trans}$ and a second rotation $\delta_{rot2}$. The turns and translations are noisy.

We model the robot’s motion using these three parameters; Yet the actual motion may have been quite different (e.g. rotating while translating)
Given $u_t = [\bar{x}_{t-1}, \bar{x}_t]^T$ we can derive $\delta_{rot1}$, $\delta_{trans}$, and $\delta_{rot2}$ via simple geometry.
Given $u_t = [\bar{x}_{t-1}, \bar{x}_t]^T$ we can derive $\delta_{rot1}$, $\delta_{trans}$, and $\delta_{rot2}$ via simple geometry.

COVERED ON BOARD
Given $u_t = [\bar{x}_{t-1}, \bar{x}_t]^T$ we can derive $\delta_{rot1}$, $\delta_{trans}$, and $\delta_{rot2}$ via simple geometry.

COVERED ON BOARD

$$\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$
Given \( u_t = [ \bar{x}_{t-1}, \bar{x}_t ]^T \) we can derive \( \delta_{rot1}, \delta_{trans}, \) and \( \delta_{rot2} \) via simple geometry.

**COVERED ON BOARD**

\[
\delta_{rot1} = \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta} \\
\delta_{trans} = \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y})^2}
\]
Given \( u_t = [ \bar{x}_{t-1}, \bar{x}_t ]^T \) we can derive \( \delta_{rot1}, \delta_{trans}, \) and \( \delta_{rot2} \) via simple geometry.

**COVERED ON BOARD**

\[
\begin{align*}
\delta_{rot1} &= \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta} \\
\delta_{trans} &= \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2} \\
\delta_{rot2} &= \bar{\theta}' - \bar{\theta} - \delta_{rot1}
\end{align*}
\]
Given $u_t = [\bar{x}_{t-1}, \bar{x}_t]^T$ we can derive $\delta_{rot1}$, $\delta_{trans}$, and $\delta_{rot2}$ via simple geometry.

COVERED ON BOARD

\[
\begin{align*}
\delta_{rot1} &= \text{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta} \\
\delta_{trans} &= \sqrt{(\bar{x} - \bar{x}')^2 + (\bar{y} - \bar{y}')^2} \\
\delta_{rot2} &= \bar{\theta}' - \bar{\theta} - \delta_{rot1}
\end{align*}
\]
We assume that these measured values are corrupted by independent noise, such that the true values are given as follows,

\[
\hat{\delta}_{\text{true}}^{\text{rot}}_1 = \delta_{\text{rot}}^1 - \epsilon_{\text{rot}}^1 \\
\hat{\delta}_{\text{true}}^{\text{trans}} = \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\hat{\delta}_{\text{true}}^{\text{rot}}_2 = \delta_{\text{rot}}^2 - \epsilon_{\text{rot}}^2
\]

where the three \( \epsilon \)'s are Normal random variables assumed to have zero mean.
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\delta_{\text{true}}^{\text{rot}1} = \delta_{\text{rot}1} - \epsilon_{\text{rot}1}
\]
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{true rot}1} &= \delta_{\text{rot}1} - \epsilon_{\text{rot}1} \\
\delta_{\text{true trans}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}}
\end{align*}
\]
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{true}^{rot1} &= \delta_{rot1} - \epsilon_{rot1} \\
\delta_{true}^{trans} &= \delta_{trans} - \epsilon_{trans} \\
\delta_{true}^{rot2} &= \delta_{rot2} - \epsilon_{rot2}
\end{align*}
\]
We assume that these measured values are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{rot}1}^{\text{true}} &= \delta_{\text{rot}1} - \epsilon_{\text{rot}1} \\
\delta_{\text{trans}}^{\text{true}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\delta_{\text{rot}2}^{\text{true}} &= \delta_{\text{rot}2} - \epsilon_{\text{rot}2}
\end{align*}
\]

where the three \( \epsilon \)'s are Normal random variables assumed to have zero mean.
We assume that these measured values are corrupted by independent noise, such that the true values are given as follows,

\[
\delta_{true}^{rot1} = \delta_{rot1} - \epsilon_{rot1} \\
\delta_{true}^{trans} = \delta_{trans} - \epsilon_{trans} \\
\delta_{true}^{rot2} = \delta_{rot2} - \epsilon_{rot2}
\]

where the three \( \epsilon \)'s are Normal random variables assumed to have zero mean.

We don’t know these true parameters
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{true rot}1} &= \delta_{\text{rot}1} - \epsilon_{\text{rot}1} \\
\delta_{\text{true trans}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\delta_{\text{true rot}2} &= \delta_{\text{rot}2} - \epsilon_{\text{rot}2}
\end{align*}
\]

where the three \(\epsilon\)'s are Normal random variables assumed to have zero mean.

We don’t know these true parameters. But given a pair of poses \(x_t\) and \(x_{t-1}\) we can compute the parameters we would expect
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{true rot}1} &= \delta_{\text{rot}1} - \epsilon_{\text{rot}1} \\
\delta_{\text{true trans}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\delta_{\text{true rot}2} &= \delta_{\text{rot}2} - \epsilon_{\text{rot}2}
\end{align*}
\]

where the three \( \epsilon \)'s are Normal random variables assumed to have zero mean.

We don’t know these true parameters. But given a pair of poses \( x_t \) and \( x_{t-1} \) we can compute the parameters we would expect

\[
x_t = [x', y', \theta']^T \quad x_{t-1} = [x, y, \theta]^T
\]
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{true}}^{\text{rot}1} &= \delta_{\text{rot}1} - \epsilon_{\text{rot}1} \\
\delta_{\text{true}}^{\text{trans}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\delta_{\text{true}}^{\text{rot}2} &= \delta_{\text{rot}2} - \epsilon_{\text{rot}2}
\end{align*}
\]

where the three \( \epsilon \)'s are Normal random variables assumed to have zero mean.

We don’t know these true parameters. But given a pair of poses \( x_t \) and \( x_{t-1} \) we can compute the parameters we would expect

\[
x_t = [x', y', \theta']^T \quad x_{t-1} = [x, y, \theta]^T
\]

\[
\hat{\delta}_{\text{rot}1} = \text{atan2}(y' - y, x' - x) - \theta
\]
We assume that these *measured values* are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{rot}1}^{\text{true}} &= \delta_{\text{rot}1} - \epsilon_{\text{rot}1} \\
\delta_{\text{trans}}^{\text{true}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\delta_{\text{rot}2}^{\text{true}} &= \delta_{\text{rot}2} - \epsilon_{\text{rot}2}
\end{align*}
\]

where the three \( \epsilon \)'s are Normal random variables assumed to have zero mean.

We don’t know these true parameters. But given a pair of poses \( x_t \) and \( x_{t-1} \) we can compute the parameters we would expect

\[
\begin{align*}
x_t &= [x', y', \theta']^T \\
x_{t-1} &= [x, y, \theta]^T
\end{align*}
\]

\[
\begin{align*}
\hat{\delta}_{\text{rot}1} &= \text{atan2}(y' - y, x' - x) - \theta \\
\hat{\delta}_{\text{trans}} &= \sqrt{(x - x')^2 + (y - y')^2}
\end{align*}
\]
We assume that these measured values are corrupted by independent noise, such that the true values are given as follows,

\[
\begin{align*}
\delta_{\text{true rot}}^1 &= \delta_{\text{rot}}^1 - \epsilon_{\text{rot}}^1 \\
\delta_{\text{true trans}} &= \delta_{\text{trans}} - \epsilon_{\text{trans}} \\
\delta_{\text{true rot}}^2 &= \delta_{\text{rot}}^2 - \epsilon_{\text{rot}}^2
\end{align*}
\]

where the three \( \epsilon \)'s are Normal random variables assumed to have zero mean.

We don't know these true parameters. But given a pair of poses \( x_t \) and \( x_{t-1} \) we can compute the parameters we would expect

\[
x_t = [x', y', \theta']^T \quad x_{t-1} = [x, y, \theta]^T
\]

\[
\begin{align*}
\hat{\delta}_{\text{rot}}^1 &= \text{atan2}(y' - y, x' - x) - \theta \\
\hat{\delta}_{\text{trans}} &= \sqrt{(x - x')^2 + (y - y')^2} \\
\hat{\delta}_{\text{rot}}^2 &= \theta' - \theta - \hat{\delta}_{\text{rot}}^1
\end{align*}
\]
In order to work out the probability of a given movement, we need to know the variance of the noise processes, $\epsilon_{rot1}$, $\epsilon_{trans}$, and $\epsilon_{rot2}$. We assume that the amount of noise is proportional to the amount of movement, as measured by odometry. We utilize the equations below to model the relationship between the amount of motion and the amount of uncertainty:

$$\sigma_{rot1}^2 = \alpha_1 \delta_{rot1}^2 + \alpha_2 \delta_{trans}^2$$

$$\sigma_{trans}^2 = \alpha_3 \delta_{trans}^2 + \alpha_4 (\delta_{rot1}^2 + \delta_{rot2}^2)$$

$$\sigma_{rot2}^2 = \alpha_1 \delta_{rot2}^2 + \alpha_2 \delta_{trans}^2$$

where the $\alpha$ parameters specify the noise characteristics of a particular robot (to be determined experimentally).

Note: This model differs from the book which relates the $\sigma$ quantities to $\hat{\delta}$ quantities.
In order to work out the probability of a given movement, we need to know the variance of the noise processes, \( \epsilon_{rot1} \), \( \epsilon_{trans} \), and \( \epsilon_{rot2} \). We assume that the amount of noise is proportional to the amount of movement, as measured by odometry.
In order to work out the probability of a given movement, we need to know the variance of the noise processes, $\epsilon_{rot1}$, $\epsilon_{trans}$, and $\epsilon_{rot2}$. We assume that the amount of noise is proportional to the amount of movement, as measured by odometry.

We utilize the equations below to model the relationship between the amount of motion and the amount of uncertainty:

\[
\begin{align*}
\sigma_{rot1}^2 &= \alpha_1 \delta_{rot1}^2 + \alpha_2 \delta_{trans}^2 \\
\sigma_{trans}^2 &= \alpha_3 \delta_{trans}^2 + \alpha_4 (\delta_{rot1}^2 + \delta_{rot2}^2) \\
\sigma_{rot2}^2 &= \alpha_1 \delta_{rot2}^2 + \alpha_2 \delta_{trans}^2
\end{align*}
\]

where the $\alpha$ parameters specify the noise characteristics of a particular robot (to be determined experimentally).

Note: This model differs from the book which relates the $\sigma$ quantities to the $\hat{\delta}$ quantities.
In order to work out the probability of a given movement, we need to know the variance of the noise processes, $\epsilon_{rot1}$, $\epsilon_{trans}$, and $\epsilon_{rot2}$. We assume that the amount of noise is proportional to the amount of movement, as measured by odometry.

We utilize the equations below to model the relationship between the amount of motion and the amount of uncertainty:

\[
\begin{align*}
\sigma^2_{rot1} &= \alpha_1 \delta^2_{rot1} + \alpha_2 \delta^2_{trans} \\
\sigma^2_{trans} &= \alpha_3 \delta^2_{trans} + \alpha_4 (\delta^2_{rot1} + \delta^2_{rot2}) \\
\sigma^2_{rot2} &= \alpha_1 \delta^2_{rot2} + \alpha_2 \delta^2_{trans}
\end{align*}
\]
In order to work out the probability of a given movement, we need to know the variance of the noise processes, $\epsilon_{\text{rot}1}$, $\epsilon_{\text{trans}}$, and $\epsilon_{\text{rot}2}$. We assume that the amount of noise is proportional to the amount of movement, as measured by odometry.

We utilize the equations below to model the relationship between the amount of motion and the amount of uncertainty:

\[
\begin{align*}
\sigma_{\text{rot1}}^2 &= \alpha_1 \delta_{\text{rot1}}^2 + \alpha_2 \delta_{\text{trans}}^2 \\
\sigma_{\text{trans}}^2 &= \alpha_3 \delta_{\text{trans}}^2 + \alpha_4 (\delta_{\text{rot1}}^2 + \delta_{\text{rot2}}^2) \\
\sigma_{\text{rot2}}^2 &= \alpha_1 \delta_{\text{rot2}}^2 + \alpha_2 \delta_{\text{trans}}^2
\end{align*}
\]

where the $\alpha$ parameters specify the noise characteristics of a particular robot (to be determined experimentally).
In order to work out the probability of a given movement, we need to know the variance of the noise processes, $\epsilon_{rot1}$, $\epsilon_{trans}$, and $\epsilon_{rot2}$. We assume that the amount of noise is proportional to the amount of movement, as measured by odometry.

We utilize the equations below to model the relationship between the amount of motion and the amount of uncertainty:

$$
\sigma^2_{rot1} = \alpha_1 \delta^2_{rot1} + \alpha_2 \delta^2_{trans}
$$
$$
\sigma^2_{trans} = \alpha_3 \delta^2_{trans} + \alpha_4 (\delta^2_{rot1} + \delta^2_{rot2})
$$
$$
\sigma^2_{rot2} = \alpha_1 \delta^2_{rot2} + \alpha_2 \delta^2_{trans}
$$

where the $\alpha$ parameters specify the noise characteristics of a particular robot (to be determined experimentally).

Note: This model differs from the book which relates the $\sigma$ quantities to the $\hat{\delta}$ quantities.
The odometry motion model computes $p(x_t | u_t, x_{t-1})$ as follows:
The odometry motion model computes $p(x_t | u_t, x_{t-1})$ as follows:

- Compute $\delta_{rot1}, \delta_{trans},$ and $\delta_{rot2}$ from odometry.
The odometry motion model computes \( p(x_t|u_t, x_{t-1}) \) as follows:

- Compute \( \delta_{\text{rot}1}, \delta_{\text{trans}}, \) and \( \delta_{\text{rot}2} \) from odometry
  - \( \bar{x}_t \) is computed by integrating wheel motion; \( \bar{x}_{t-1} \) is simply the same result from the last time step
The odometry motion model computes $p(x_t|u_t, x_{t-1})$ as follows:

- Compute $\delta_{rot1}, \delta_{trans},$ and $\delta_{rot2}$ from odometry
  - $\bar{x}_t$ is computed by integrating wheel motion; $\bar{x}_{t-1}$ is simply the same result from the last time step
- Compute $\hat{\delta}_{rot1}, \hat{\delta}_{trans},$ and $\hat{\delta}_{rot2}$ for a specific pair of $x_t$ and $x_{t-1}$
The odometry motion model computes $p(x_t|u_t, x_{t-1})$ as follows:

- Compute $\delta_{rot1}, \delta_{trans},$ and $\delta_{rot2}$ from odometry
  - $\bar{x}_t$ is computed by integrating wheel motion; $\bar{x}_{t-1}$ is simply the same result from the last time step
- Compute $\hat{\delta}_{rot1}, \hat{\delta}_{trans},$ and $\hat{\delta}_{rot2}$ for a specific pair of $x_t$ and $x_{t-1}$
- Assuming for the moment that the “hat” parameters are true, determine the probability of obtaining $\delta_{rot1}, \delta_{trans},$ and $\delta_{rot2}$
The odometry motion model computes $p(x_t|u_t, x_{t-1})$ as follows:

- Compute $\delta_{rot1}, \delta_{trans}$, and $\delta_{rot2}$ from odometry
  - $\bar{x}_t$ is computed by integrating wheel motion; $\bar{x}_{t-1}$ is simply the same result from the last time step
- Compute $\hat{\delta}_{rot1}, \hat{\delta}_{trans}$, and $\hat{\delta}_{rot2}$ for a specific pair of $x_t$ and $x_{t-1}$
- Assuming for the moment that the “hat” parameters are true, determine the probability of obtaining $\delta_{rot1}, \delta_{trans}$, and $\delta_{rot2}$
The odometry motion model computes $p(x_t|u_t, x_{t-1})$ as follows:

- Compute $\delta_{rot1}, \delta_{trans},$ and $\delta_{rot2}$ from odometry
  - $\bar{x}_t$ is computed by integrating wheel motion; $\bar{x}_{t-1}$ is simply the same result from the last time step
- Compute $\hat{\delta}_{rot1}, \hat{\delta}_{trans},$ and $\hat{\delta}_{rot2}$ for a specific pair of $x_t$ and $x_{t-1}$
- Assuming for the moment that the “hat” parameters are true, determine the probability of obtaining $\delta_{rot1}, \delta_{trans},$ and $\delta_{rot2}$
This probability is computed as follows:
This probability is computed as follows:

\[ p_1 = \text{prob}(\delta_{\text{rot}1} - \hat{\delta}_{\text{rot}1}, \sigma_{\text{rot}1}^2) \]
This probability is computed as follows:

\[ p_1 = \text{prob}(\delta_{\text{rot}1} - \hat{\delta}_{\text{rot}1}, \sigma_{\text{rot}1}^2) \]

\[ p_2 = \text{prob}(\delta_{\text{trans}} - \hat{\delta}_{\text{trans}}, \sigma_{\text{trans}}^2) \]
This probability is computed as follows:

\[ p_1 = \text{prob}(\delta_{\text{rot}1} - \hat{\delta}_{\text{rot}1}, \sigma_{\text{rot}1}^2) \]

\[ p_2 = \text{prob}(\delta_{\text{trans}} - \hat{\delta}_{\text{trans}}, \sigma_{\text{trans}}^2) \]

\[ p_3 = \text{prob}(\delta_{\text{rot}2} - \hat{\delta}_{\text{rot}2}, \sigma_{\text{rot}2}^2) \]
This probability is computed as follows:

\[ p_1 = \text{prob}(\delta_{rot1} - \hat{\delta}_{rot1}, \sigma_{rot1}^2) \]
\[ p_2 = \text{prob}(\delta_{trans} - \hat{\delta}_{trans}, \sigma_{trans}^2) \]
\[ p_3 = \text{prob}(\delta_{rot2} - \hat{\delta}_{rot2}, \sigma_{rot2}^2) \]

\[ p(x_t | u_t, x_{t-1}) = p_1 \cdot p_2 \cdot p_3 \]
This probability is computed as follows:

\[ p_1 = \text{prob}(\delta_{\text{rot}1} - \hat{\delta}_{\text{rot}1}, \sigma_{\text{rot}1}^2) \]
\[ p_2 = \text{prob}(\delta_{\text{trans}} - \hat{\delta}_{\text{trans}}, \sigma_{\text{trans}}^2) \]
\[ p_3 = \text{prob}(\delta_{\text{rot}2} - \hat{\delta}_{\text{rot}2}, \sigma_{\text{rot}2}^2) \]

\[ p(x_t | u_t, x_{t-1}) = p_1 \cdot p_2 \cdot p_3 \]

where \( \text{prob}(v, \sigma^2) \) returns the probability density for value \( v \) of a zero-mean Normal distribution with variance \( \sigma^2 \).
This probability is computed as follows:

\[ p_1 = \text{prob}(\delta_{rot1} - \hat{\delta}_{rot1}, \sigma^2_{rot1}) \]
\[ p_2 = \text{prob}(\delta_{trans} - \hat{\delta}_{trans}, \sigma^2_{trans}) \]
\[ p_3 = \text{prob}(\delta_{rot2} - \hat{\delta}_{rot2}, \sigma^2_{rot2}) \]

\[ p(x_t|u_t, x_{t-1}) = p_1 \cdot p_2 \cdot p_3 \]

where \(\text{prob}(\nu, \sigma^2)\) returns the probability density for value \(\nu\) of a zero-mean Normal distribution with variance \(\sigma^2\). The product of probabilities \(p_1 \cdot p_2 \cdot p_3\) is the joint probability under the assumption that the three noise processes are independent.
The shape of \( p(x_t|u_t, x_{t-1}) \) depends on the \( \alpha_i \) parameters, which should be chosen to match (or exceed) the robot’s actuator and odometry errors.
The shape of $p(x_t|u_t, x_{t-1})$ depends on the $\alpha_i$ parameters, which should be chosen to match (or exceed) the robot’s actuator and odometry errors.

**Figure 5.8**  The odometry motion model, for different noise parameter settings.
Measurement Model: \( p(z_t | x_t) \)

We must determine the probability of observing \( z_t \) given a pose \( x_t \).
Measurement Model: \( p(z_t | x_t) \)

We must determine the probability of observing \( z_t \) given a pose \( x_t \). The measurement model should incorporate all of the sensor’s various failure modes:

\( \text{Figure 6.1} \) (a) Typical ultrasound scan of a robot in its environment. (b) A misreading in ultrasonic sensing. This effect occurs when firing a sonar signal towards a reflective surface at an angle \( \alpha \) that exceeds half the opening angle of the sensor.
Measurement Model: \( p(z_t | x_t) \)

We must determine the probability of observing \( z_t \) given a pose \( x_t \). The measurement model should incorporate all of the sensor’s various failure modes:

**Figure 6.1** (a) Typical ultrasound scan of a robot in its environment. (b) A misreading in ultrasonic sensing. This effect occurs when firing a sonar signal towards a reflective surface at an angle \( \alpha \) that exceeds half the opening angle of the sensor.
Measurement Model: $p(z_t|x_t)$

We must determine the probability of observing $z_t$ given a pose $x_t$. The measurement model should incorporate all of the sensor’s various failure modes:

![Diagram](image)

**Figure 6.1** (a) Typical ultrasound scan of a robot in its environment. (b) A misreading in ultrasonic sensing. This effect occurs when firing a sonar signal towards a reflective surface at an angle $\alpha$ that exceeds half the opening angle of the sensor.

The measurement model depends heavily on the map, $m$
A particular sensor observation $z_t$ may be composed of a number of different sensor readings.
A particular sensor observation $z_t$ may be composed of a number of different sensor readings. The identity of these individual readings are indicated in superscript,
A particular sensor observation $z_t$ may be composed of a number of different sensor readings. The identity of these individual readings are indicated in superscript,

$$z_t = \left\{ z_t^1, \ldots, z_t^K \right\}$$
A particular sensor observation $z_t$ may be composed of a number of different sensor readings. The identity of these individual readings are indicated in superscript,

$$z_t = \left\{ z_t^1, \ldots, z_t^K \right\}$$

where there are $K$ individual sensors.
A particular sensor observation $z_t$ may be composed of a number of different sensor readings. The identity of these individual readings are indicated in superscript,

$$z_t = \left\{ z_t^1, \ldots, z_t^K \right\}$$

where there are $K$ individual sensors.

We assume that individual sensors are independent,
A particular sensor observation $z_t$ may be composed of a number of different sensor readings. The identity of these individual readings are indicated in superscript,

$$z_t = \{ z_t^1, \ldots, z_t^K \}$$

where there are $K$ individual sensors.

We assume that individual sensors are independent,

$$p(z_t|x_t) = \prod_{k=1}^{K} p(z_t^k|x_t)$$
The beam model for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors.
The **beam model** for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z_t^k$ is a range.
The beam model for range-finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z_t^k$ is a range.

$p(z_t^k|x_t)$ is computed as a mixture of four distributions.
The beam model for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z_t^k$ is a range.

$p(z_t^k|x_t)$ is computed as a mixture of four distributions. These four distributions come from the probability of the following conditions:
The **beam model** for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z_t^k$ is a range.

$p(z_t^k|x_t)$ is computed as a mixture of four distributions. These four distributions come from the probability of the following conditions:

1. The sensor has measured the correct range, with some noise
The **beam model** for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z_t^k$ is a range.

$p(z_t^k|x_t)$ is computed as a mixture of four distributions. These four distributions come from the probability of the following conditions:

1. The sensor has measured the correct range, with some noise

Let the true range be $z_t^{k*}$
The **beam model** for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z_t^k$ is a range.

$p(z_t^k|x_t)$ is computed as a mixture of four distributions. These four distributions come from the probability of the following conditions:

1. **The sensor has measured the correct range, with some noise**

Let the true range be $z_t^{k*}$. The random variable $z_t^k$ is assumed to be Normally distributed with mean $z_t^{k*}$ and standard deviation $\sigma_{hit}$.
The **beam model** for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z^k_t$ is a range.

$p(z^k_t|x_t)$ is computed as a mixture of four distributions. These four distributions come from the probability of the following conditions:

1. **The sensor has measured the correct range, with some noise**

   Let the true range be $z^k_t^*$. The random variable $z^k_t$ is assumed to be Normally distributed with mean $z^k_t^*$ and standard deviation $\sigma_{hit}$.

   Thus, if we know $z^k_t^*$ and $\sigma_{hit}$ we can calculate $p_{hit}(z^k_t|x_t)$
The **beam model** for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z_t^k$ is a range.

$p(z_t^k|x_t)$ is computed as a mixture of four distributions. These four distributions come from the probability of the following conditions:

1. **The sensor has measured the correct range, with some noise**

Let the true range be $z_t^{k*}$. The random variable $z_t^k$ is assumed to be Normally distributed with mean $z_t^{k*}$ and standard deviation $\sigma_{\text{hit}}$.

Thus, if we know $z_t^{k*}$ and $\sigma_{\text{hit}}$ we can calculate $p_{\text{hit}}(z_t^k|x_t)$. But how do we determine $z_t^{k*}$?
The **beam model** for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z_t^k$ is a range.

$p(z_t^k|x_t)$ is computed as a mixture of four distributions. These four distributions come from the probability of the following conditions:

1. **The sensor has measured the correct range, with some noise**

Let the true range be $z_t^{k\ast}$. The random variable $z_t^k$ is assumed to be Normally distributed with mean $z_t^{k\ast}$ and standard deviation $\sigma_{hit}$.

Thus, if we know $z_t^{k\ast}$ and $\sigma_{hit}$ we can calculate $p_{hit}(z_t^k|x_t)$. But how do we determine $z_t^{k\ast}$?

We **cast a ray** from position $x_t$ in the map in the direction that our sensor is facing.
The beam model for range finding sensors is a good model for both laser rangefinders and ultrasonic sensors. We will assume that $z_t^k$ is a range.

\[ p(z_t^k|\mathbf{x}_t) \] is computed as a mixture of four distributions. These four distributions come from the probability of the following conditions:

1. The sensor has measured the correct range, with some noise

Let the true range be $z_t^{k*}$. The random variable $z_t^k$ is assumed to be Normally distributed with mean $z_t^{k*}$ and standard deviation $\sigma_{hit}$.

Thus, if we know $z_t^{k*}$ and $\sigma_{hit}$ we can calculate $p_{hit}(z_t^k|\mathbf{x}_t)$. But how do we determine $z_t^{k*}$?

We cast a ray from position $\mathbf{x}_t$ in the map in the direction that our sensor is facing. When this ray hits its first obstacle we set $z_t^{k*}$ to the distance it has travelled.
We may wish to restrict the distribution to have zero probability after the sensor’s maximum range $z_{max}$, as we know the sensor will not produce a larger value.
We may wish to restrict the distribution to have zero probability after the sensor’s maximum range $z_{max}$, as we know the sensor will not produce a larger value. If this is done it would also be necessary to normalize the distribution to ensure its sum is 1.
We may wish to restrict the distribution to have zero probability after the sensor’s maximum range $z_{\text{max}}$, as we know the sensor will not produce a larger value. If this is done it would also be necessary to normalize the distribution to ensure its sum is 1.

(a) Gaussian distribution $p_{\text{hit}}$

$$p\left(z_t^k \mid x_t, m\right)$$
2. Unexpected objects

The map may omit many objects (e.g. people). The presence of such objects will tend to reduce the reported range. The probability of an object interposing itself between the robot and a mapped part of the environment decreases with range. This probability can be modelled as an exponential distribution, truncated at the true range.
2. Unexpected objects

The map may omit many objects (e.g. people)
2. Unexpected objects

The map may omit many objects (e.g. people). The presence of such object will tend to *reduce* the reported range.
2. Unexpected objects

The map may omit many objects (e.g. people). The presence of such object will tend to *reduce* the reported range. The probability of an object interposing itself between the robot and a mapped part of the environment decreases with range.
2. Unexpected objects

The map may omit many objects (e.g. people). The presence of such object will tend to *reduce* the reported range. The probability of an object interposing itself between the robot and a mapped part of the environment decreases with range. This probability can be modelled as an exponential distribution, truncated at the true range.
2. Unexpected objects

The map may omit many objects (e.g. people). The presence of such object will tend to \textit{reduce} the reported range. The probability of an object interposing itself between the robot and a mapped part of the environment decreases with range. This probability can be modelled as an exponential distribution, truncated at the true range.

\begin{center}
(b) Exponential distribution \( p_{\text{short}} \)
\end{center}

\begin{align*}
p(z_t^k | x_t, m) & \quad \text{for } z_t^k \leq z_t^{k*} \\
& \quad \text{and } z_t^{k*} < z_{\text{max}}
\end{align*}
3. Missed objects

The sensor may miss an object altogether. Or the emitted beam may experience specular reflection and never return to the sensor. This possibility is modelled by a "spike" in probability at the maximum range of the sensor $z_{\text{max}}$. 
3. Missed objects

The sensor may miss an object altogether.
3. Missed objects

The sensor may miss an object altogether. Or the emitted beam may experience specular reflection and never return to the sensor.
3. Missed objects

The sensor may miss an object altogether. Or the emitted beam may experience specular reflection and never return to the sensor. This possibility is modelled by a “spike” in probability at the maximum range of the sensor $z_{\text{max}}$. 
3. Missed objects

The sensor may miss an object altogether. Or the emitted beam may experience specular reflection and never return to the sensor. This possibility is modelled by a “spike” in probability at the maximum range of the sensor $z_{\text{max}}$.

(c) Uniform distribution $p_{\text{max}}$

$$p(z_t^k \mid x_t, m)$$

$z_t^* \quad z_{\text{max}}$
4. Unexplainable measurements

Sometimes sensors produce inexplicable measurements, for no apparent reason. We "model" such occurrences by a uniform distribution.
4. Unexplainable measurements

Sometimes sensors produce inexplicable measurements, for no apparent reason.
4. Unexplainable measurements

Sometimes sensors produce inexplicable measurements, for no apparent reason. We “model” such occurrences by a uniform distribution.
4. Unexplainable measurements

Sometimes sensors produce inexplicable measurements, for no apparent reason. We “model” such occurrences by a uniform distribution.

(d) Uniform distribution $p_{\text{rand}}$

\[
p(z_t^k \mid x_t, m)
\]

$z_t^k$ $z_{\text{max}}$
The following shows all four of these densities individually:

(a) Gaussian distribution

\[ p(z_k^* | x_t, m) \]

(b) Exponential distribution

\[ p(z_k^* | x_t, m) \]

(c) Uniform distribution

\[ p(z_{max}^* | x_t, m) \]

(d) Uniform distribution

\[ p(z_{max}^* | x_t, m) \]
The following shows all four of these densities individually:

(a) Gaussian distribution \(p_{\text{hit}}\)

\[ p(Z^k_t | x_t, m) \]

(b) Exponential distribution \(p_{\text{short}}\)

\[ p(Z^k_t | x_t, m) \]

(c) Uniform distribution \(p_{\text{max}}\)

\[ p(Z^k_t | x_t, m) \]

(d) Uniform distribution \(p_{\text{rand}}\)

\[ p(Z^k_t | x_t, m) \]

**Figure 6.3** Components of the range finder sensor model. In each diagram the horizontal axis corresponds to the measurement \(Z^k_t\), the vertical to the likelihood.
A weighted combination of the four densities gives $p(z^k_t|x_t)$:
A weighted combination of the four densities gives $p(z_t^k | x_t)$:

![Graph showing a distribution with peaks at $z_t^k$ and $z_{max}$]
Figure 6.7 Probabilistic model of perception: (a) Laser range scan, projected into a previously acquired map \( m \). (b) The likelihood \( p(z_t \mid x_t, m) \), evaluated for all positions \( x_t \) and projected into the map (shown in gray). The darker a position, the larger \( p(z_t \mid x_t, m) \).
Figure 6.7  Probabilistic model of perception: (a) Laser range scan, projected into a previously acquired map $m$. (b) The likelihood $p(z_t \mid x_t, m)$, evaluated for all positions $x_t$ and projected into the map (shown in gray). The darker a position, the larger $p(z_t \mid x_t, m)$. 
Grid localization is the direct application of Bayes filter to mobile robot localization using a discrete grid (usually 3-D) as the belief representation.
Grid localization is the direct application of Bayes filter to mobile robot localization using a discrete grid (usually 3-D) as the belief representation.

- Uses the **discrete Bayes filter** (integrals replaced with summation).
Grid localization is the direct application of Bayes filter to mobile robot localization using a discrete grid (usually 3-D) as the belief representation.

- Uses the **discrete Bayes filter** (integrals replaced with summation)
- Can be applied on a fine-grained grid or on a topological decomposition of the belief state
Grid localization is the direct application of Bayes filter to mobile robot localization using a discrete grid (usually 3-D) as the belief representation.

- Uses the discrete Bayes filter (integrals replaced with summation).

- Can be applied on a fine-grained grid or on a topological decomposition of the belief state.

- Fine-grained approaches yield good results but with a high computational cost.
Grid localization is the direct application of Bayes filter to mobile robot localization using a discrete grid (usually 3-D) as the belief representation.

- Uses the discrete Bayes filter (integrals replaced with summation)
- Can be applied on a fine-grained grid or on a topological decomposition of the belief state
- Fine-grained approaches yield good results but with a high computational cost
- Has the ability to track multiple hypotheses
**Grid localization** is the direct application of Bayes filter to mobile robot localization using a discrete grid (usually 3-D) as the belief representation.

- Uses the **discrete Bayes filter** (integrals replaced with summation).
- Can be applied on a fine-grained grid or on a topological decomposition of the belief state.
- Fine-grained approaches yield good results but with a high computational cost.
- Has the ability to track multiple hypotheses.
- The measurement model is usually defined on raw sensor values → feature extraction not required.
Grid localization using a fine-grained metric decomposition. Each picture depicts the position of the robot in the hallway along with its belief \( \text{bel}(x_t) \), represented by a histogram over a grid.
For robots operating in the plane we require a 3-D probability cube to represent our belief in the robot’s pose $x_t = [x, y, \theta]^T$. 

Grid Environment

Figure 8.2 Example of a fixed-resolution grid over the robot pose variables $x$, $y$, and $\theta$. Each grid cell represents a robot pose in the environment. Different orientations of the robot correspond to different planes in the grid (shown are only three orientations).
For robots operating in the plane we require a 3-D probability cube to represent our belief in the robot’s pose $x_t = [x, y, \theta]^T$.

Figure 8.2  Example of a fixed-resolution grid over the robot pose variables $x$, $y$, and $\theta$. Each grid cell represents a robot pose in the environment. Different orientations of the robot correspond to different planes in the grid (shown are only three orientations).
For robots operating in the plane we require a 3-D probability cube to represent our belief in the robot’s pose $x_t = [x, y, \theta]^T$

Each grid cell contains the probability that the robot has the corresponding pose.
Grid localization requires a map so that the measurement model can be applied
Grid localization requires a map so that the measurement model can be applied. The following is a typical occupancy grid map:
Grid localization requires a map so that the measurement model can be applied. The following is a typical occupancy grid map:

![Occupancy grid map of the 1994 AAAI mobile robot competition arena.](image)

**Figure 8.8** Occupancy grid map of the 1994 AAAI mobile robot competition arena.
- Typical resolution of probability cube: 15 cm in x and y, 5° in θ
• Typical resolution of probability cube: 15 cm in $x$ and $y$, 5° in $\theta$
• With greater (i.e. more fine grained) resolution the accuracy increases, but so does the computation time
- Typical resolution of probability cube: 15 cm in $x$ and $y$, $5^\circ$ in $\theta$
- With greater (i.e. more fine grained) resolution the accuracy increases, but so does the computation time
- If the resolution is very coarse, transitions between cells become increasingly unlikely for slow-moving robots (e.g. with 1 m cell size and 10 cm movements, the probability of transitioning between cells is very remote)
• Typical resolution of probability cube: 15 cm in $x$ and $y$, $5^\circ$ in $\theta$
• With greater (i.e. more fine grained) resolution the accuracy increases, but so does the computation time
• If the resolution is very coarse, transitions between cells become increasingly unlikely for slow-moving robots (e.g. with 1 m cell size and 10 cm movements, the probability of transitioning between cells is very remote)
  • This can be accounted for by inflating the amount of noise in the motion model, but this decreases accuracy
• Typical resolution of probability cube: 15 cm in $x$ and $y$, 5° in $\theta$

• With greater (i.e. more fine grained) resolution the accuracy increases, but so does the computation time

• If the resolution is very coarse, transitions between cells become increasingly unlikely for slow-moving robots (e.g. with 1 m cell size and 10 cm movements, the probability of transitioning between cells is very remote)
  
  • This can be accounted for by inflating the amount of noise in the motion model, but this decreases accuracy

• A naive implementation of grid localization can be quite slow:
- Typical resolution of probability cube: 15 cm in $x$ and $y$, $5^\circ$ in $\theta$
- With greater (i.e. more fine grained) resolution the accuracy increases, but so does the computation time
- If the resolution is very coarse, transitions between cells become increasingly unlikely for slow-moving robots (e.g. with 1 m cell size and 10 cm movements, the probability of transitioning between cells is very remote)
  - This can be accounted for by inflating the amount of noise in the motion model, but this decreases accuracy
- A naive implementation of grid localization can be quite slow:
  - Application of the motion model involves iterating over all $x_t$, and for each $x_t$ one must iterate over all $x_{t-1}$
Typical resolution of probability cube: 15 cm in x and y, 5° in θ

With greater (i.e. more fine grained) resolution the accuracy increases, but so does the computation time

If the resolution is very coarse, transitions between cells become increasingly unlikely for slow-moving robots (e.g. with 1 m cell size and 10 cm movements, the probability of transitioning between cells is very remote)

- This can be accounted for by inflating the amount of noise in the motion model, but this decreases accuracy

A naive implementation of grid localization can be quite slow:

- Application of the motion model involves iterating over all $x_t$, and for each $x_t$ one must iterate over all $x_{t-1}$
• Typical resolution of probability cube: 15 cm in x and y, 5° in θ

• With greater (i.e. more fine grained) resolution the accuracy increases, but so does the computation time

• If the resolution is very coarse, transitions between cells become increasingly unlikely for slow-moving robots (e.g. with 1 m cell size and 10 cm movements, the probability of transitioning between cells is very remote)
  
  • This can be accounted for by inflating the amount of noise in the motion model, but this decreases accuracy

• A naive implementation of grid localization can be quite slow:
  
  • Application of the motion model involves iterating over all $x_t$, and for each $x_t$ one must iterate over all $x_{t-1}$. For a $n \times n \times n$ probability cube this is a $O(n^6)$ operation!
- Typical resolution of probability cube: 15 cm in x and y, 5° in θ
- With greater (i.e. more fine grained) resolution the accuracy increases, but so does the computation time
- If the resolution is very coarse, transitions between cells become increasingly unlikely for slow-moving robots (e.g. with 1 m cell size and 10 cm movements, the probability of transitioning between cells is very remote)
  - This can be accounted for by inflating the amount of noise in the motion model, but this decreases accuracy
- A naive implementation of grid localization can be quite slow:
  - Application of the motion model involves iterating over all $x_t$, and for each $x_t$ one must iterate over all $x_{t-1}$. For a $n \times n \times n$ probability cube this is a $O(n^6)$ operation!
  - Application of the measurement model iterates over all $x_t$, and for each $x_t$ one must iterate over all $k$ sensor values. For range sensors, each scan point requires a ray casting operation.
Computational considerations

There are a number of ways of speeding up grid localization:

- Reduce frequency of updates.
  - Note: If we reduce the frequency of applications of the motion model, we must integrate the motion for a longer period in between updates.

- Decrease grid resolution.
  - Problems: decreases accuracy, may require exaggerated noise in motion model.

- Selective updating:
  - Based on probability threshold: Update only those cells of the belief cube which have a probability greater than some fixed threshold.
  - Problem: if localization errors occur (e.g. the kidnapped robot problem) the robot may end up in an area which is not updated.

- Apply updates in local neighbourhood: When updating a cell, restrict the space of possible prior poses to those centred around that cell (other variants on this scheme are possible).
Computational considerations

There are a number of ways of speeding up grid localization:

- Reduce frequency of updates.
Computational considerations

There are a number of ways of speeding up grid localization:

- Reduce frequency of updates.
  - Note: If we reduce the frequency of applications of the motion model, we must integrate the motion for a longer period in between updates.

- Decrease grid resolution.
  - Problems: decreases accuracy, may require exaggerated noise in motion model.

- Selective updating:
  - Based on probability threshold: Update only those cells of the belief cube which have a probability greater than some fixed threshold.
  - Problem: if localization errors occur (e.g. the kidnapped robot problem) the robot may end up in an area which is not updated.

- Apply updates in local neighbourhood: When updating a cell, restrict the space of possible prior poses to those centred around that cell (other variants on this scheme are possible).
There are a number of ways of speeding up grid localization:

- Reduce frequency of updates.
  - Note: If we reduce the frequency of applications of the motion model, we must integrate the motion for a longer period in between updates.

- Decrease grid resolution.

Problems: decreases accuracy, may require exaggerated noise in motion model.

Selective updating:
- Based on probability threshold: Update only those cells of the belief cube which have a probability greater than some fixed threshold.
  - Problem: if localization errors occur (e.g. the kidnapped robot problem) the robot may end up in an area which is not updated.

Apply updates in local neighbourhood: When updating a cell, restrict the space of possible prior poses to those centred around that cell (other variants on this scheme are possible).
There are a number of ways of speeding up grid localization:

- **Reduce frequency of updates.**
  - Note: If we reduce the frequency of applications of the motion model, we must integrate the motion for a longer period in between updates.
- **Decrease grid resolution.**
  - Problems: decreases accuracy, may require exaggerated noise in motion model.
There are a number of ways of speeding up grid localization:

- Reduce frequency of updates.
  - Note: If we reduce the frequency of applications of the motion model, we must integrate the motion for a longer period in between updates.

- Decrease grid resolution.
  - Problems: decreases accuracy, may require exaggerated noise in motion model.

- Selective updating:
Computational considerations

There are a number of ways of speeding up grid localization:

- **Reduce frequency of updates.**
  - Note: If we reduce the frequency of applications of the motion model, we must integrate the motion for a longer period in between updates.

- **Decrease grid resolution.**
  - Problems: decreases accuracy, may require exaggerated noise in motion model.

- **Selective updating:**
  - Based on probability threshold: Update only those cells of the belief cube which have a probability greater than some fixed threshold.
Computational considerations

There are a number of ways of speeding up grid localization:

- **Reduce frequency of updates.**
  
  Note: If we reduce the frequency of applications of the motion model, we must integrate the motion for a longer period in between updates.

- **Decrease grid resolution.**
  
  Problems: decreases accuracy, may require exaggerated noise in motion model.

- **Selective updating:**
  
  Based on probability threshold: Update only those cells of the belief cube which have a probability greater than some fixed threshold.

  Problem: if localization errors occur (e.g. the kidnapped robot problem) the robot may end up in an area which is not updated.
Computational considerations

There are a number of ways of speeding up grid localization:

- **Reduce frequency of updates.**
  - Note: If we reduce the frequency of applications of the motion model, we must integrate the motion for a longer period in between updates.

- **Decrease grid resolution.**
  - Problems: decreases accuracy, may require exaggerated noise in motion model.

- **Selective updating:**
  - Based on probability threshold: Update only those cells of the belief cube which have a probability greater than some fixed threshold.
    - Problem: if localization errors occur (e.g. the kidnapped robot problem) the robot may end up in an area which is not updated.
  - Apply updates in local neighbourhood: When updating a cell, restrict the space of possible prior poses to those centred around that cell (*other variants on this scheme are possible*)
The measurement model can also be accelerated:
The measurement model can also be accelerated:

- Pre-caching results during initialization:
The measurement model can also be accelerated:

- Pre-caching results during initialization:
  - For each possible 3D pose, cast rays for each sensor to determine beforehand the expected ranges for each pose. Store these ranges in a table for later look-up.
The measurement model can also be accelerated:

- Pre-caching results during initialization:
  - For each possible 3D pose, cast rays for each sensor to determine beforehand the expected ranges for each pose. Store these ranges in a table for later look-up.
  - Pre-compute the p.d.f.’s for all possible ranges.
The measurement model can also be accelerated:

- **Pre-caching results during initialization:**
  - For each possible 3D pose, cast rays for each sensor to determine beforehand the expected ranges for each pose. Store these ranges in a table for later look-up.
  - Pre-compute the p.d.f.’s for all possible ranges.

- **Reduce the number of sensors used.**
An example of grid localization
An example of grid localization. It is important to note that the probabilities in the grid have been projected from 3-D to 2-D.
An example of grid localization. It is important to note that the probabilities in the grid have been projected from 3-D to 2-D. The distribution of belief over $\theta$ is very important, but it is not shown below.
An example of grid localization. It is important to note that the probabilities in the grid have been projected from 3-D to 2-D. The distribution of belief over $\theta$ is very important, but it is not shown below.

Figure 8.9  (a) Data set (odometry and sonar range scans) collected in the environment shown in Figure 8.8. This data set is sufficient for global localization using the grid localization. The beliefs at the points marked “A,” “B” and “C” are shown in (b), (c), and (d).
An example of grid localization. It is important to note that the probabilities in the grid have been projected from 3-D to 2-D. The distribution of belief over $\theta$ is very important, but it is not shown below.

Figure 8.9  (a) Data set (odometry and sonar range scans) collected in the environment shown in Figure 8.8. This data set is sufficient for global localization using the grid localization. The beliefs at the points marked “A,” “B” and “C” are shown in (b), (c), and (d).
*Probabilistic Robotics.*
MIT Press.