Unit 4: Localization Part 2
Introduction to Markov Localization

Computer Science 4766/6912

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A robot that can solve the kidnapped robot problem is able to recover from errors much more readily than one that cannot.

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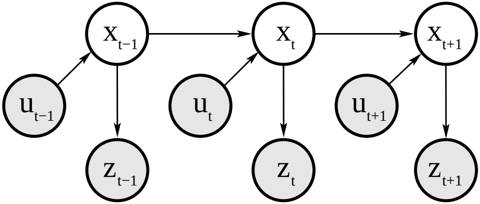

Figure 2.2 The dynamic Bayes network that characterizes the evolution of controls, states, and measurements.

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For this assumption to hold, the state vector must include a *complete* description of all objects within the environment (inluding a complete map; a description of all people/robots/animals in the environment and their complete state vectors; etc...).

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Thus, this assumption is generally untrue in practise, but we utilize it nonetheless as it renders the localization problem tractable.

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This is known as the **motion model**.

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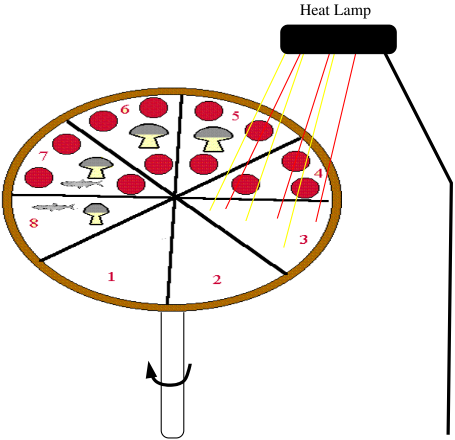

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$$bel(x_t = i) = \sum_{j=1}^{8} p(x_t = i|x_{t-1} = j)bel(x_{t-1} = j)$$

We need a *motion model* for $p(x_t = i|x_{t-1} = j)$. Lets say that the turning mechanism has a 0.5 probability of turning the pizza by one slice; a 0.25 probability of turning by two slices; and a 0.25 probability of not turning it at all. We assume all turns are in the positive direction.

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$$p(x_t = i|x_{t-1} = j) = \begin{cases} 0.25 & \text{for } i = j \\ 0.5 & \text{for } i = j \oplus 1 \\ & \\ & \end{cases}$$

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where $\oplus$ indicates addition with wrap-around (e.g. $7 \oplus 2 = 1$)

Assume that $bel(x_0)$ is 1 for $x_0 = 1$ and 0 otherwise

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t	1	2	3	4	5	6	7	8
0	1	0	0	0	0	0	0	0
1	0.25	0.5	0.25	0	0	0	0	0

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t	1	2	3	4	5	6	7	8
0	1	0	0	0	0	0	0	0
1	0.25	0.5	0.25	0	0	0	0	0
2	0.0625	0.25	0.375	0.25	0.0625	0	0	0

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1	0.25	0.5	0.25	0	0	0	0	0
2	0.0625	0.25	0.375	0.25	0.0625	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

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t	1	2	3	4	5	6	7	8
0	1	0	0	0	0	0	0	0
1	0.25	0.5	0.25	0	0	0	0	0
2	0.0625	0.25	0.375	0.25	0.0625	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
60	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

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	i							
t	1	2	3	4	5	6	7	8
0	1	0	0	0	0	0	0	0
1	0.25	0.5	0.25	0	0	0	0	0
2	0.0625	0.25	0.375	0.25	0.0625	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
60	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

A peak in $bel(x_t)$ at the most likely position persists for some time, but our uncertainty about which slice is under the lamp only grows with time—eventually leading to global uncertainty (i.e. a uniform distribution).

Back to the Bayes filter...

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The measurement update: equation (2)

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$$\eta = \frac{1}{\int p(z_t|x_t)\overline{bel}(x_t)dx_t}$$

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Typically, the measurement model requires a map of the environment so that we can determine how likely it was to observe z_t at position x_t.

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The mushroom map:

slice i	1	2	3	4	5	6	7	8
Mu?	Yes	Yes	Yes	Yes	No	No	No	No
$p(z_t = Yes \mid x_t = i)$	0.9	0.9	0.9	0.9	0.1	0.1	0.1	0.1

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The mushroom map:

slice i	1	2	3	4	5	6	7	8
Mu?	Yes	Yes	Yes	Yes	No	No	No	No
$p(z_t = Yes \vert x_t = i)$	0.9	0.9	0.9	0.9	0.1	0.1	0.1	0.1
$p(z_t = No \vert x_t = i)$	0.1	0.1	0.1	0.1	0.9	0.9	0.9	0.9

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$p(z_t = No \mid x_t = i)$	0.1	0.1	0.1	0.1	0.9	0.9	0.9	0.9

The bottom two rows give the measurement model—the mushroom detector is correct 90% of the time

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t	$\overline{bel}(x_t)$

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1	0.25	0.5	0.25	0	0	0	0	0

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1	0.25	0.5	0.25	0	0	0	0	0

To apply the measurement update, we need to know z_t. Assume $z_t = Yes$. $p(z_t = Yes | x_t = 1) = 0.9$, so

t	almost $bel(x_t)$							
1	0.225	0.45	0.225	0	0	0	0	0

Assume again that we know the heat lamp is initially on slice 1. The application of the prediction step yields,

t	$\overline{bel}(x_t)$							
1	0.25	0.5	0.25	0	0	0	0	0

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We then have to normalize this 'almost correct' belief

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t	$bel(x_t)$							
1	0.25	0.5	0.25	0	0	0	0	0
2	$\overline{bel}(x_t)$							

We continue to update our belief; Assume $z_2 = z_3 = Yes$,

t	$bel(x_t)$							
1	0.25	0.5	0.25	0	0	0	0	0
2	$\overline{bel}(x_t)$							
	0.0625	0.25	0.375	0.25	0.0625	0	0	0

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	0.0625	0.25	0.375	0.25	0.0625	0	0	0
	$bel(x_t)$							
	0.0662	0.2647	0.3971	0.2647	0.0074	0	0	0

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t	$bel(x_t)$							
1	0.25	0.5	0.25	0	0	0	0	0
2	$\overline{bel}(x_t)$							
	0.0625	0.25	0.375	0.25	0.0625	0	0	0
	$bel(x_t)$							
	0.0662	0.2647	0.3971	0.2647	0.0074	0	0	0
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1	0.25	0.5	0.25	0	0	0	0	0
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	0.0625	0.25	0.375	0.25	0.0625	0	0	0
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	0.0662	0.2647	0.3971	0.2647	0.0074	0	0	0
3	$\overline{bel}(x_t)$							
	0.0165	0.0993	0.2482	0.3309	0.2335	0.0699	0.0018	0

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	0.0165	0.0993	0.2482	0.3309	0.2335	0.0699	0.0018	0
	$bel(x_t)$							
	0.0227	0.1362	0.3405	0.4540	0.0356	0.0107	0.0003	0

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2	$\overline{bel}(x_t)$							
	0.0625	0.25	0.375	0.25	0.0625	0	0	0
	$bel(x_t)$							
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Notice how the measurements now reduce uncertainty.

Derivation of Bayes Filter

We will derive Bayes filter below

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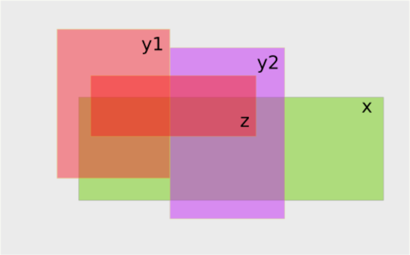

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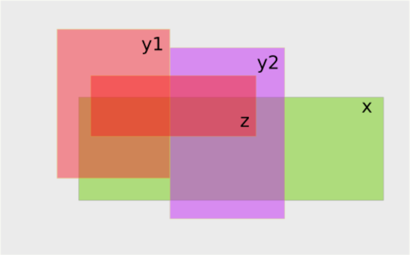

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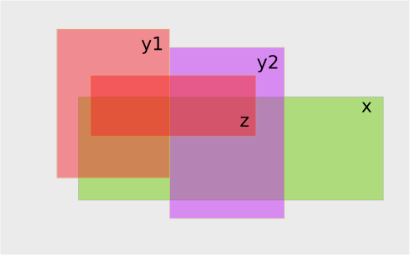

The Law of Total Probability requires mutually exclusive and exhaustive events y_i. Hence:

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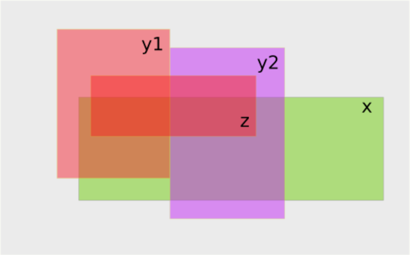

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$$p(x) \neq \sum_i p(x|y_i)p(y_i)$$

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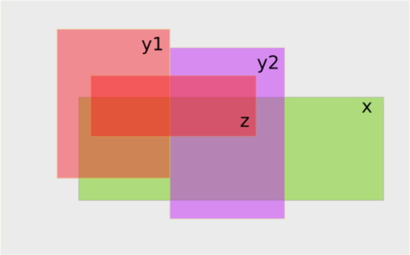

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\begin{aligned}
p(x) &\neq \sum_i p(x|y_i)p(y_i) \\
p(x|z) &= \sum_i p(x|y_i, z)p(y_i|z)
\end{aligned}
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This is the motion model.

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We can recognize this as $bel(x_{t-1})$ which is the input to Bayes filter.

$$p(x_t|z_{1:t-1}, u_{1:t}) = \int p(x_t|x_{t-1}, u_t)p(x_{t-1}|z_{1:t-1}, u_{1:t})dx_{t-1}$$

Consider the second factor,

$$p(x_{t-1}|z_{1:t-1}, u_{1:t})$$

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This is equation (1) of Bayes filter.

The above analysis shows that Bayes filter appropriately computes $bel(x_t)$ if the input $bel(x_{t-1})$ is correct

The above analysis shows that Bayes filter appropriately computes $bel(x_t)$ if the input $bel(x_{t-1})$ is correct. If we assume that the initial belief $bel(x_0)$ at time $t = 0$ was correct, then the correctness of Bayes filter follows by induction.

References

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