Navigation

- Providing robots with the requisite sensors, actuators, and algorithms to allow them to navigate in general environments is a big job; Four major subtasks are...
  - Perception: extract meaningful data from sensors
  - Localization: determine the robot’s position
  - Cognition / Planning: decide on what actions to take
  - Motion control: control the actuators
- Localization answers the question “Where am I?”

Why is Localization Difficult?

- Doesn’t odometry solve the localization problem?
  - Localization by odometry (i.e. integrating wheel rotation) is subject to cumulative error
- Doesn’t GPS solve the localization problem?
  - GPS is enormously helpful in some applications, but has disadvantages for others:
    - GPS will be unavailable in a variety of situations:
      - Indoors
      - In obstructed areas
      - Underwater
      - In space (or on other worlds)
      - If the U.S. decides to make it unavailable
    - Insufficiently accurate for localizing smaller robots (such as the “body-navigating nanorobots of the future” [Siegwart and Nourbakhsh, 2004])
Issues

- Sensor noise:
  - Noise from proprioceptive sensors such as wheel encoders makes localization w.r.t. last known position difficult
  - Noise from exteroceptive sensors makes localization w.r.t. a map difficult
- Perceptual aliasing: the sensor data obtained at one location is indistinguishable from the data obtained at another location
- Noise introduced by movement:
  - Systematic errors: e.g. Differences between wheel radii
  - Non-Systematic errors: e.g. Wheel slippage

Map Representation

- In this part of the course we will assume the existence of a map
- Map used to compare with the robot’s current sensory state
  - Can be supplied to the robot or,
  - Built autonomously — SLAM (Simultaneous Localization and Mapping)
- Choice of map representation depends on precision required, available sensors, and computational constraints

A continuous representation represents all mapped objects in continuous-space.

e.g. represent map as the set of infinite lines through object boundaries [Tomatis et al., 2003]

An occupancy grid imposes a grid upon the world, with grid cells corresponding to free space left empty; and those for objects filled

Cons: inexact, size of map grows quickly

Cons: Requires storage proportional to the number of objects
An adaptive decomposition can improve the storage efficiency of an occupancy grid obtained by recursively splitting occupied grid cells into sub-cells.

A topological representation represents the world as a graph, with nodes used to represent positions, and edges passable paths between positions.

Topological representations have two main requirements:
1. A method to detect current position (node)
2. A method of travelling between nodes (e.g. beacon aiming, corridor centreing, wall following, visual homing,...)

Visual features particularly useful for (1)
- pros:
  - The local navigation strategies required for (2) can be quite simple
  - This representation is lightweight (low memory cost) and well-suited for planning
- cons:
  - Difficulty in maintaining a consistent density of nodes
  - Subject to perceptual aliasing problems

Belief Representation

We will represent a robot’s belief in its position as a probability distribution:
- There are a number of ways to represent probability distributions
  - Continuous or discrete
  - Single-hypothesis or multiple-hypothesis
    - Single-hypothesis: The robot believes it is at one position with some margin for error
    - Multiple-hypothesis: The robot can represent the belief that it could be at a number of possible positions
(a) Continuous, single-hypothesis (Gaussian)
(b) Continuous, multiple-hypothesis (mixture of Gaussians)
(c) Discrete, multiple-hypothesis
(d) Discrete, multiple-hypothesis (for a topological map)

Single-hypothesis belief:
- (a) in previous slide
- Very efficient to update
- May not be suitable if starting from a position of global uncertainty (i.e. no clue as to where you are)

Multiple-hypothesis beliefs:
- (b), (c), or (d) on previous slide
- A sampled representation of a continuous distribution is also possible (Monte Carlo Localization)
- Representation is more powerful, but updates can be very expensive

Example of multiple-hypothesis belief tracking using a discrete representation,

Path of the robot

Belief states at positions 2, 3 and 4

Large initial uncertainty is reduced by new observations

Probability Review

To understand probabilistic localization we will have to review some concepts from probability. We start with \textit{conditional probability}.

Given that some event $B$ has occurred, the probability $p(A|B)$ gives the conditional probability that $A$ has also occurred.

To illustrate, consider the following probability pizza:

Assume that we pick a random slice of pizza with uniform probability...
The definition of conditional probability:

\[ p(A|B) = \frac{p(A \land B)}{p(B)} \]

\[ p(\text{Pepperoni}) = \frac{5}{8} \]
\[ p(\text{Anchovies}) = \frac{3}{8} \]
\[ p(\text{Anchovies}|\text{Pepperoni}) = \frac{1}{5} \]

Bayes Rule

If we write the definition of conditional probability as \( p(B|A) \) we can obtain the following well-known rule (DERIVATION COVERED ON BOARD),

\[ p(A|B) = \frac{p(B|A)p(A)}{p(B)} \]

This is called Bayes rule and allows us to calculate a conditional probability (a.k.a. prior probability) from its “inverse”
e.g. What is \( p(\text{Pepperoni}|\text{Anchovies})? \)

\[ p(\text{Pepperoni}|\text{Anchovies}) = \frac{p(\text{Anchovies}|\text{Pepperoni})p(\text{Pepperoni})}{p(\text{Anchovies})} = \frac{1}{3} \]

Theorem of Total Probability

\[ p(\text{Mushrooms}) = p(\text{Mu}) = \frac{1}{2} \]

If \( A_1, \ldots, A_n \) are mutually exclusive and exhaustive events, the theorem of total probability says that:

\[ p(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i) \]

\[ p(\text{Mu}) = p(\text{Mu}|\text{Slice1})p(\text{Slice1}) + p(\text{Mu}|\text{Slice2})p(\text{Slice2}) + \cdots \]
\[ = 0 \cdot \frac{1}{8} + 0 \cdot \frac{1}{8} + 0 \cdot \frac{1}{8} + 0 \cdot \frac{1}{8} + 0 \cdot \frac{1}{8} + 1 \cdot \frac{1}{8} + 1 \cdot \frac{1}{8} + 1 \cdot \frac{1}{8} + 1 \cdot \frac{1}{8} = 1/2 \]

Independence

Two events \( A \) and \( B \) are independent iff their joint probability is equal to the product of their individual probabilities

\[ p(A \land B) = p(A)p(B) \]

For the pizza on the left, events \( \text{Pe} \) and \( \text{An} \) are independent: COVERED

However, for the pizza on the right these events are not independent: COVERED

ON BOARD
Conditional Independence

Sometimes two events can be independent, but only if some other event is known to have occurred. This is known as **conditional independence** and is expressed by the following relation,

\[ p(A \land B \mid C) = p(A \mid C)p(B \mid C) \]

For this pizza, if we know that a slice contains mushrooms, the event that it contains anchovies is independent from the event that it contains pepperoni.

References
