Unit 4: Localization Part 1
Maps, Beliefs, and Probability Review

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Introduction

- Navigation
- Why is Localization Difficult?
- Issues

Map Representation

Belief Representation

Probability Review
Providing robots with the requisite sensors, actuators, and algorithms to allow them to navigate in general environments is a big job; Four major subtasks are...

- Perception: extract meaningful data from sensors
- Localization: determine the robot’s position
- Cognition / Planning: decide on what actions to take
- Motion control: control the actuators

Localization answers the question “Where am I?”
Why is Localization Difficult?

Doesn’t odometry solve the localization problem?

- Localization by odometry (i.e. integrating wheel rotation) is subject to cumulative error

Doesn’t GPS solve the localization problem?

- GPS is enormously helpful in some applications, but has disadvantages for others:
  - GPS will be unavailable in a variety of situations:
    - Indoors
    - In obstructed areas
    - Underwater
    - In space (or on other worlds)
    - If the U.S. decides to make it unavailable
  - Insufficiently accurate for localizing smaller robots (such as the “body-navigating nanorobots of the future” [Siegwart and Nourbakhsh, 2004])
Issues

- **Sensor noise:**
  - Noise from proprioceptive sensors such as wheel encoders makes localization w.r.t. last known position difficult
  - Noise from exteroceptive sensors makes localization w.r.t. a map difficult

- **Perceptual aliasing:** the sensor data obtained at one location is indistinguishable from the data obtained at another location

- **Noise introduced by movement:**
  - Systematic errors: e.g. Differences between wheel radii
  - Non-Systematic errors: e.g. Wheel slippage
In this part of the course we will assume the existence of a map

Map used to compare with the robot's current sensory state
  - Can be supplied to the robot or,
  - Built autonomously — SLAM (Simultaneous Localization and Mapping)

Choice of map representation depends on precision required, available sensors, and computational constraints
A **continuous representation** represents all mapped objects in continuous-space.

e.g. represent map as the set of infinite lines through object boundaries [Tomatis et al., 2003]

![Diagram of an office and its representation as lines through object boundaries.](image)

**Fig. 4.** An office of the institute (a) and the lines representing it in the local metric map (b). The black segments permit to see the correspondence between the two figures.

**Cons:** Requires storage proportional to the number of objects
An **occupancy grid** imposes a grid upon the world, with grid cells corresponding to free space left empty; and those for objects filled

Cons: inexact, size of map grows quickly
An adaptive decomposition can improve the storage efficiency of an occupancy grid

Obtained by recursively splitting occupied grid cells into sub-cells
A **topological representation** represents the world as a graph, with **nodes** used to represent positions, and **edges** passable paths between positions.
Topological representations have two main requirements:

1. A method to detect current position (node)
2. A method of travelling between nodes (e.g. beacon aiming, corridor centreing, wall following, visual homing,...)

Visual features particularly useful for (1)

Pros:
- The local navigation strategies required for (2) can be quite simple
- This representation is lightweight (low memory cost) and well-suited for planning

Cons:
- Difficulty in maintaining a consistent density of nodes
- Subject to perceptual aliasing problems
We will represent a robot’s belief in its position as a probability distribution.

There are a number of ways to represent probability distributions:

- Continuous or discrete
- Single-hypothesis or multiple-hypothesis
  - Single-hypothesis: The robot believes it is at one position with some margin for error
  - Multiple-hypothesis: The robot can represent the belief that it could be at a number of possible positions
(a) Continuous, single-hypothesis (Gaussian)
(b) Continuous, multiple-hypothesis (mixture of Gaussians)
(c) Discrete, multiple-hypothesis
(d) Discrete, multiple-hypothesis (for a topological map)
• Single-hypothesis belief:
  • (a) in previous slide
  • Very efficient to update
  • May not be suitable if starting from a position of global uncertainty (i.e. no clue as to where you are)

• Multiple-hypothesis beliefs:
  • (b), (c), or (d) on previous slide
  • A sampled representation of a continuous distribution is also possible (Monte Carlo Localization)
  • Representation is more powerful, but updates can be very expensive
Example of multiple-hypothesis belief tracking using a discrete representation,

Path of the robot

Belief states at positions 2, 3 and 4

Large initial uncertainty is reduced by new observations
To understand probabilistic localization we will have to review some concepts from probability. We start with \textbf{conditional probability}.

Given that some event $B$ has occurred, the probability $p(A|B)$ gives the conditional probability that $A$ has also occurred.

To illustrate, consider the following probability pizza:

Assume that we pick a random slice of pizza with uniform probability...
\[ p(\text{Pepperoni}) = \frac{5}{8} \]

\[ p(\text{Anchovies}) = \frac{3}{8} \]

\[ p(\text{Anchovies} | \text{Pepperoni}) = \frac{1}{5} \]

**The definition of conditional probability:**

\[ p(A | B) = \frac{p(A \land B)}{p(B)} \]
Bayes Rule

If we write the definition of conditional probability as $p(B|A)$ we can obtain the following well-known rule (DERIVATION COVERED ON BOARD),

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

This is called Bayes rule and allows us to calculate a conditional probability (a.k.a. prior probability) from its “inverse”

e.g. What is $p(\text{Pepperoni}|\text{Anchovies})$?

$$p(\text{Pe}|\text{An}) = \frac{p(\text{An}|\text{Pe})p(\text{Pe})}{p(\text{An})} = \frac{1}{3}$$
**Theorem of Total Probability**

If $A_1, \ldots, A_n$ are mutually exclusive and exhaustive events, the theorem of total probability says that:

$$p(B) = \sum_{i=1}^{n} P(B|A_i)P(A_i)$$

$$p(Mu) = p(Mu|Slice1)p(Slice1) + p(Mu|Slice2)p(Slice2) + \cdots$$

$$= 0 \cdot \frac{1}{8} + 0 \cdot \frac{1}{8} + 0 \cdot \frac{1}{8} + 0 \cdot \frac{1}{8} + 1 \cdot \frac{1}{8} + 1 \cdot \frac{1}{8} + 1 \cdot \frac{1}{8} + 1 \cdot \frac{1}{8} = \frac{1}{2}$$
Independence

Two events $A$ and $B$ are independent iff their joint probability is equal to the product of their individual probabilities

$$p(A \land B) = p(A)p(B)$$

For the pizza on the left, events $Pe$ and $An$ are independent: COVERED ON BOARD

However, for the pizza on the right these events are not independent: COVERED ON BOARD
Conditional Independence

Sometimes two events can be independent, but only if some other event is known to have occurred. This is known as **conditional independence** and is expressed by the following relation,

\[ p(A \land B \mid C) = p(A \mid C)p(B \mid C) \]

For this pizza, if we know that a slice contains mushrooms, the event that it contains anchovies is independent from the event that it contains pepperoni,
References

*Introduction to Autonomous Mobile Robots.*
MIT Press.

Hybrid simultaneous localization and map building: a natural integration of topological and metric.