Unit 4: Localization Part 1
Maps, Beliefs, and Probability Review

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

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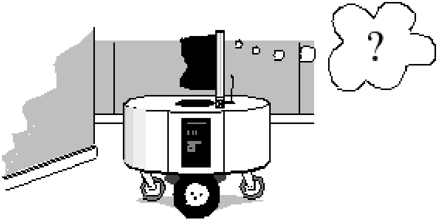

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  - Insufficiently accurate for localizing smaller robots (such as the "body-navigating nanorobots of the future" [Siegwart and Nourbakhsh, 2004])

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  - Non-Systematic errors: e.g. Wheel slippage

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- Choice of map representation depends on precision required, available sensors, and computational constraints

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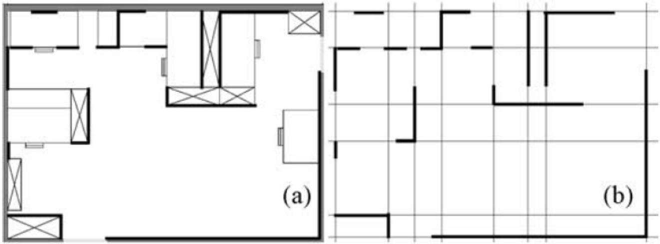

Fig. 4. An office of the institute (a) and the lines representing it in the local metric map (b). The black segments permit to see the correspondence between the two figures.

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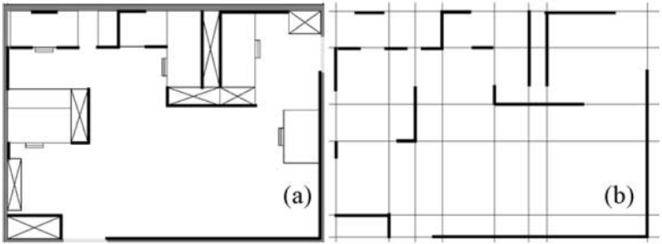

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Cons: Requires storage proportional to the number of objects

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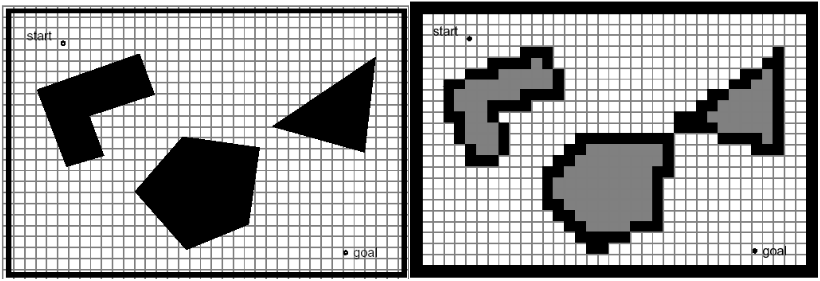

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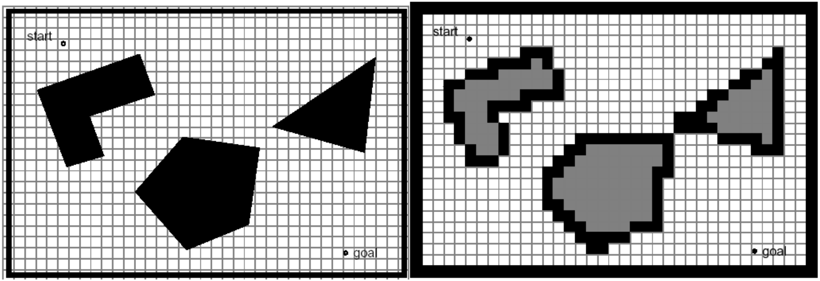

Cons: inexact, size of map grows quickly

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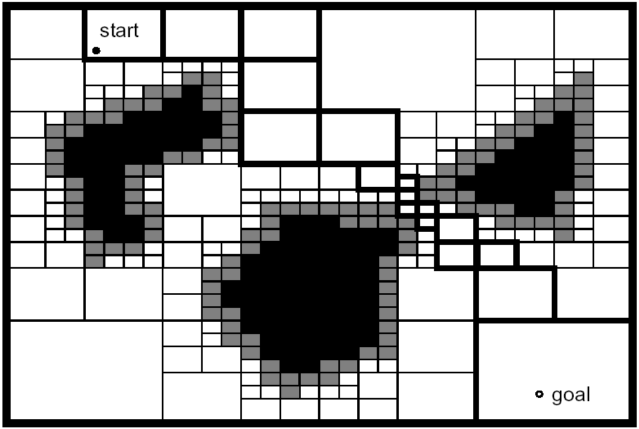

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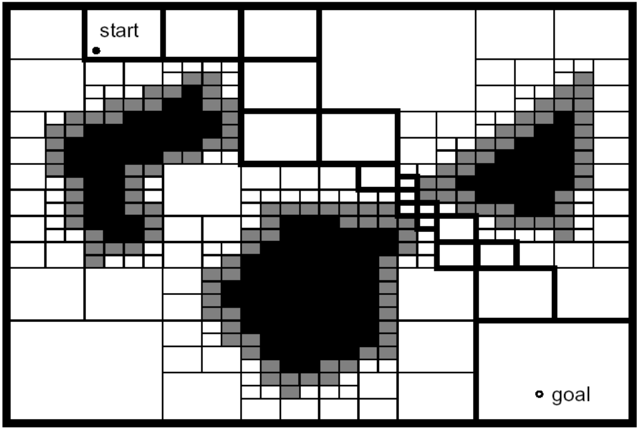

Obtained by recursively splitting occupied grid cells into sub-cells

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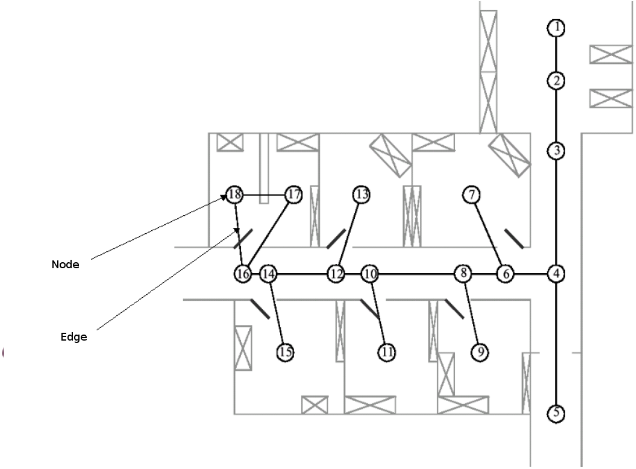

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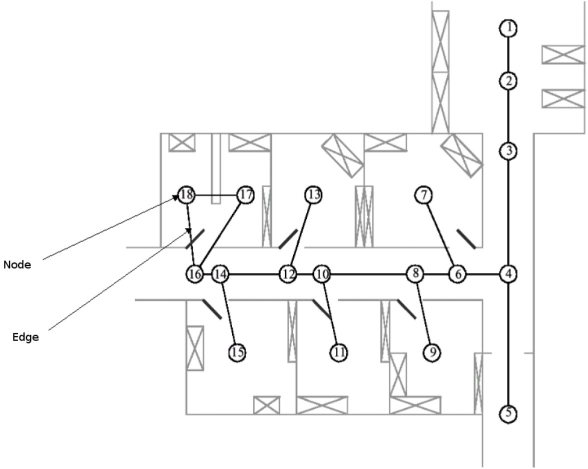

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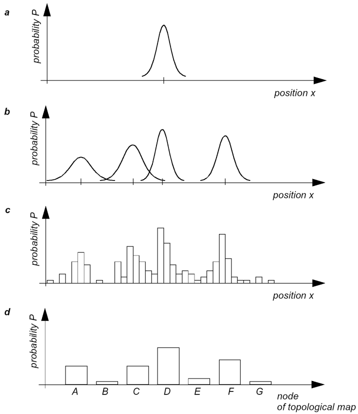
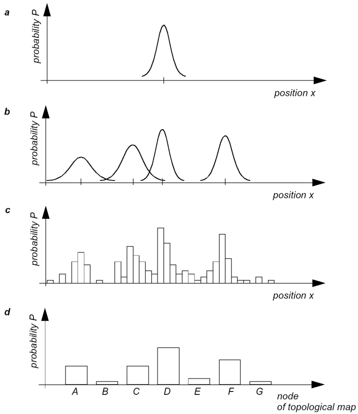


(a) Continuous, single-hypothesis (Gaussian)



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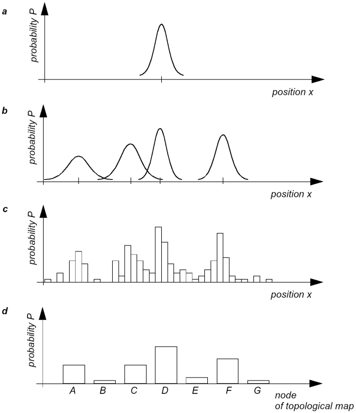
(b) Continuous, multiple-hypothesis (mixture of Gaussians)



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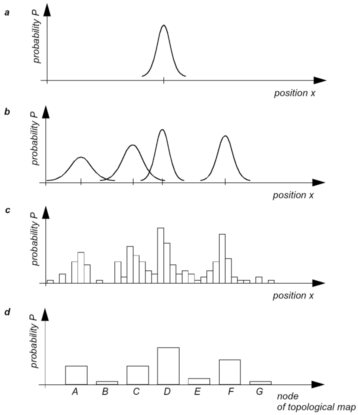

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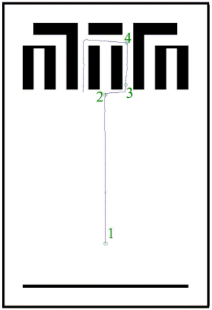
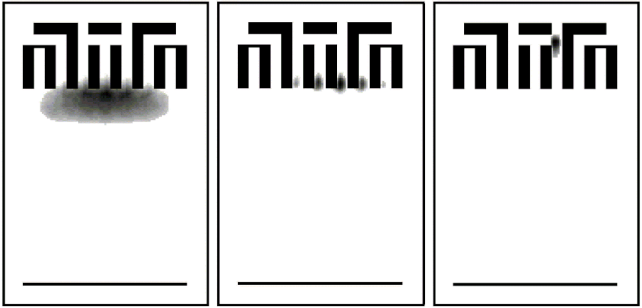


Path of the robot



Belief states at positions 2, 3 and 4

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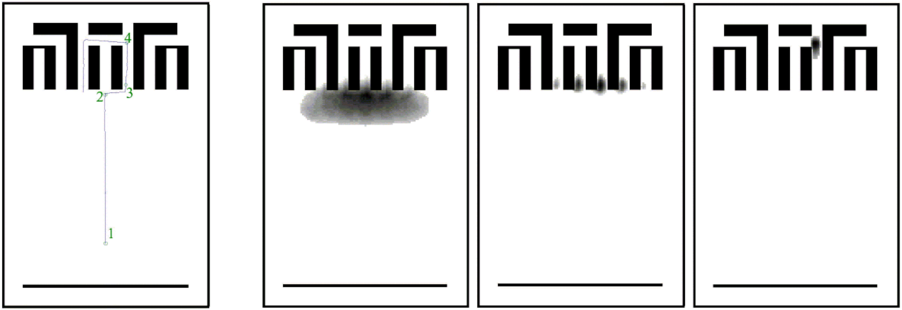

Path of the robot *Belief states at positions 2, 3 and 4*

Large initial uncertainty is reduced by new observations

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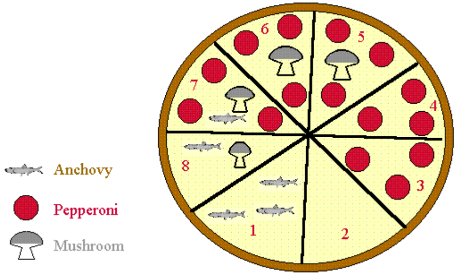

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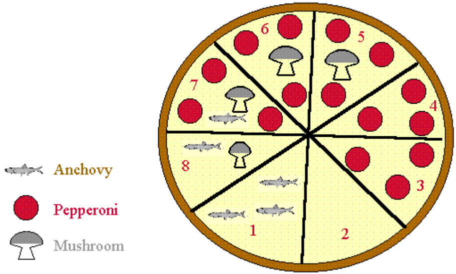
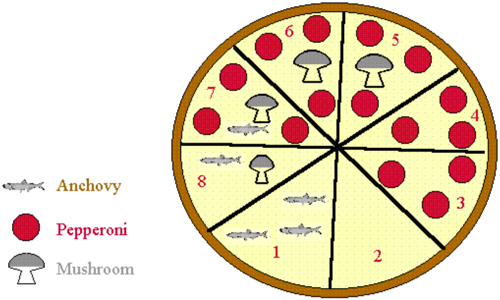
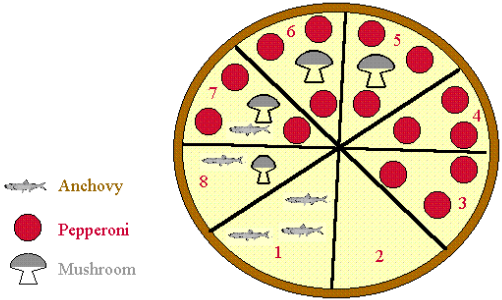


Assume that we pick a random slice of pizza with uniform probability...


Anchovy
Pepperoni
Mushroom
1
2
3
4
5
6
7
8



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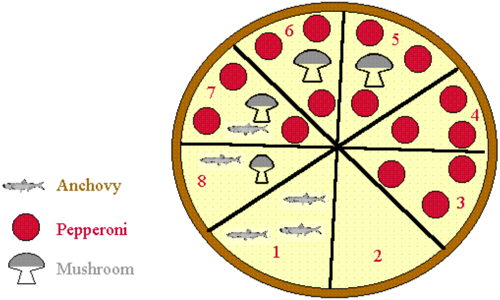

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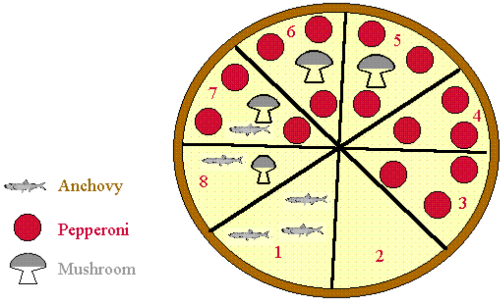

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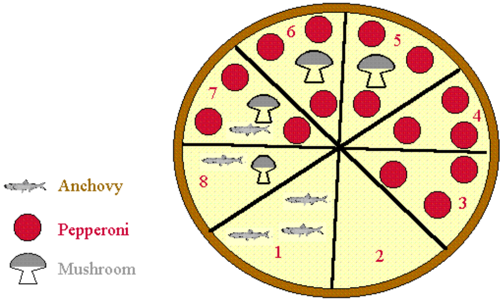

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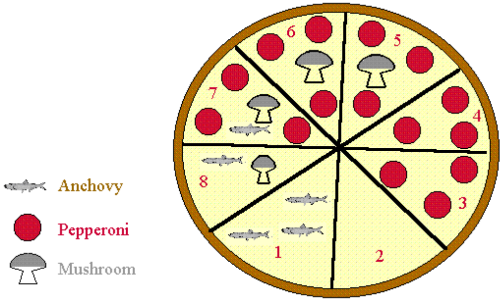

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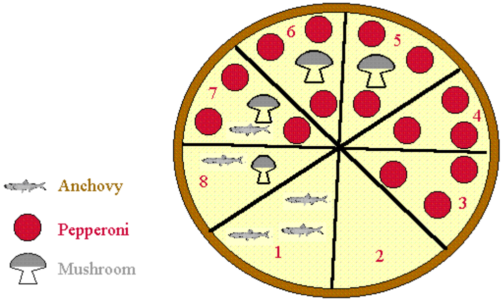
$p(Anchovies|Pepperoni) =$



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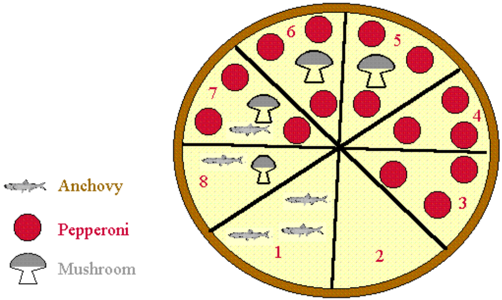

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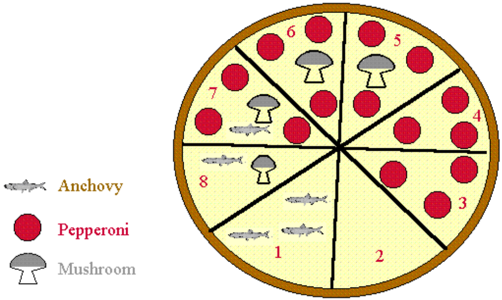

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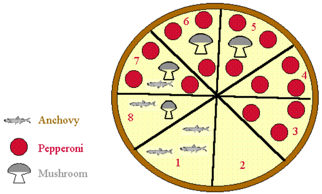

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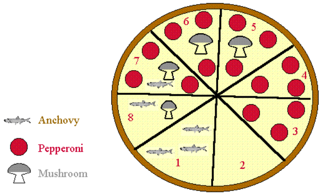

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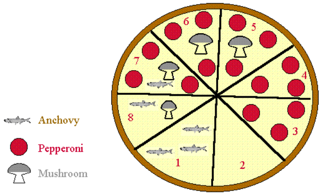
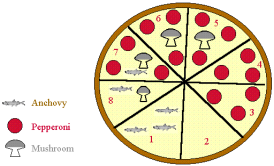


$$p(Pe|An) = \frac{p(An|Pe)p(Pe)}{p(An)} = \frac{1}{3}$$

Theorem of Total Probability



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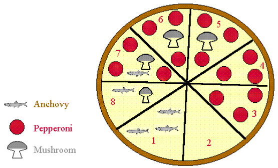

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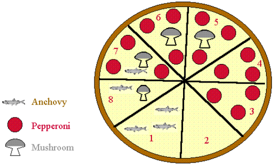

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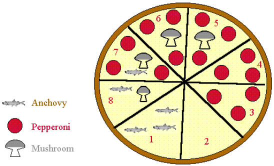

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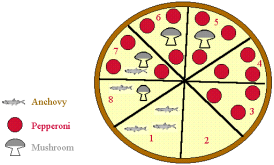

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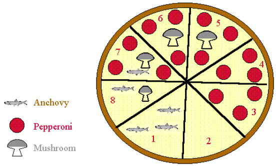

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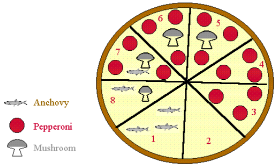

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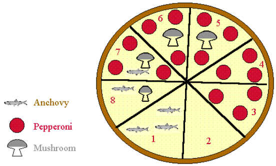

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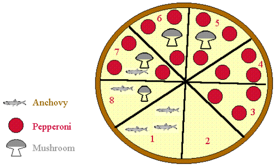

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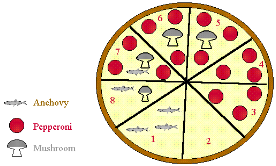

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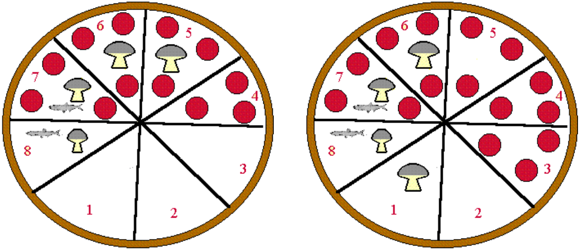

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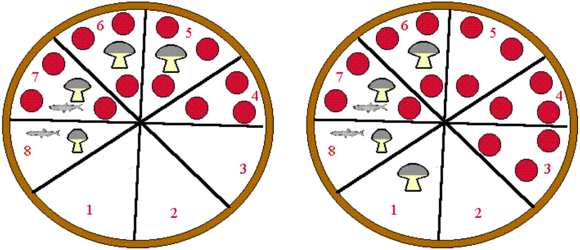

However, for the pizza on the right these events are **not** independent: COVERED ON BOARD

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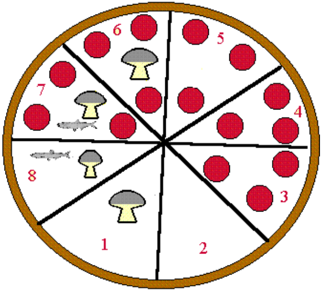

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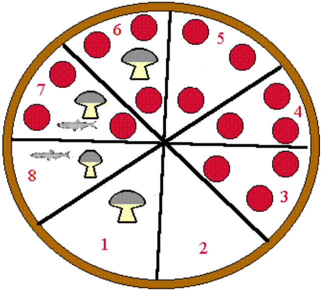

References

Siegwart, R. and Nourbakhsh, I. (2004).
Introduction to Autonomous Mobile Robots.
MIT Press.

Tomatis, N., Nourbakhsh, I., and Siegwart, R. (2003).
Hybrid simultaneous localization and map building: a natural integration of topological and metric.
Robotics and Autonomous Systems, 44:3–14.