1 Manoeuvrability

2 Motion Control

- Trajectory Following
- Closed-Loop Control
- Two-step Controller
- Smooth Controller 1
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Manoeuvrability: Degree of Mobility

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ICR’s for Various Wheel Configurations

Left: For a differential-drive the two zero-motion lines are coincident; thus, the ICR is constrained only to lie somewhere on that line.

Centre: For an Ackerman configuration (approximated by modern cars) the two rear wheels give only one zero-motion line; to prevent slipping, the two front wheels must be steered such that their zero-motion lines intersect the rear line at a common point.

Right: A degenerate configuration; there is no ICR; if there is no slipping, there is also no movement.
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To determine the degree of mobility we count the number of independent sliding constraints.
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Define a matrix $C$ that encodes the wheel direction component of the sliding constraint equations for all wheels
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  \( rank = \text{number of independent rows or columns (have to be equal)} \)
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  \]

  \[\text{rank } [C] = 1\]

- Turning bicycle:

\[
C = \begin{bmatrix}
-1 & 0 \\
-1 & \frac{\sqrt{2}}{2} \\
\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\
\end{bmatrix}
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The maximum rank of an $N \times 3$ matrix is $\delta_m = 3 - \text{rank} [C]$

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• Differential-drive: $\delta_m = 2$
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• Determining $\delta_m$ is an important part of determining how manoeuvrable a robot is; However, the fact that some wheels are steerable should also be considered...
We define a robot’s **degree of steerability**, $\delta_s$, as the number of *independently steerable wheels* that yield a valid ICR.
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We define a robot’s **degree of manoeuvrability** as follows,

$$\delta_M \equiv \delta_m + \delta_s$$
Degrees of Manoeuvrability, Mobility, and Steerability for Various Configurations

*Omnidirectional*
- $\delta_M = 3$
- $\delta_m = 3$
- $\delta_s = 0$

*Differential*
- $\delta_M = 2$
- $\delta_m = 2$
- $\delta_s = 0$

*Omni-Steer*
- $\delta_M = 3$
- $\delta_m = 2$
- $\delta_s = 1$

*Tricycle*
- $\delta_M = 2$
- $\delta_m = 1$
- $\delta_s = 1$

*Two-Steer*
- $\delta_M = 3$
- $\delta_m = 1$
- $\delta_s = 2$
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  • increased complexity and expense
  • reduced accuracy for dead reckoning
  • reduced ground clearance for powered versions
  • standard wheels can passively counteract lateral forces; more efficient and stable for high-speed turns
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Now consider the trajectory of the Two-Steer
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During time intervals 1-2 and 3-4 the robot was doing nothing but steering its wheels; the omnidirectional robot could transition between segments of the trajectory without any delay; both robots take the same path but the trajectories (path + time dimension) differ.
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Motion Control: Trajectory Following

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Open-Loop System:

The input (a.k.a. reference) is first converted by the input transducer to the form required by the controller. The process or plant carries out the core function of the system (e.g. the furnace in a heating system, motors in a robot). The output (a.k.a. the controlled variable) differs from its desired value because of the two disturbances.

- Open-loop heating system: The controller is an electronic amplifier. Disturbance 1 is noise in the amplifier's output. Disturbance 2 might be variations in temperature due to the furnace itself.
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Closed-Loop System:

In a closed-loop system there is an output transducer or sensor which converts the output into the form used by the controller. e.g. Position can be converted to an electrical signal by a potentiometer. The first summing junction subtracts the output signal from the input. This is the error signal.
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Closed-loop systems compensate for disturbances through feedback. If the actuating signal is zero then the output is correct and the plant does not need to be driven. Otherwise, the actuating signal describes how different the output is from what it should be. This drives the plant to correct this difference.

While open-loop systems fail to correct for disturbances or changes in the environment, they will tend to be simpler and cheaper than closed-loop systems. Thus, there is a trade-off to consider between them.
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Closed-Loop Control

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We need to determine how to set the robot’s forward speed \( v(t) \) and rotational speed \( \omega(t) \).
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We need to determine how to set the robot’s forward speed \( v(t) \) and rotational speed \( \omega(t) \)

For a differential-drive robot we have the following,

\[
\begin{align*}
  v(t) &= \dot{x}_R = \frac{r(\dot{\phi}_r + \dot{\phi}_l)}{2} \\
  \omega(t) &= \dot{\theta} = \frac{r(\dot{\phi}_r - \dot{\phi}_l)}{2l}
\end{align*}
\]
If we can obtain $g_R$ then we can apply some control function $f$ to get $v(t)$, the forward velocity component, and $\omega(t)$, the angular velocity component.
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The control function should drive the robot such that,

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\lim_{t \to \infty} g_R(t) = [0 \ 0]^T
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If we can obtain $g_R$ then we can apply some control function $f$ to get $v(t)$, the forward velocity component, and $\omega(t)$, the angular velocity component.

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The control function should drive the robot such that,

$$\lim_{t \to \infty} g_R(t) = [0 \ 0]^T$$

which just means that the robot will eventually reach the goal
We are given $g_I$; How do we determine $g_R$?

$g_R = R_{cw}(\theta)(g_I - [x \\ y]^T)$

(We should use the $2 \times 2$ rotation matrix here)

Clearly we need $[x \\ y]^T$; How do we get that?
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COVERED ON BOARD
...Some Details

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Clearly we need $[x y]^T$; How do we get that?

- Odometry (previously covered), or by using a map (to be covered)
Two-step Controller

- We break the problem into two steps:

\[
\begin{align*}
\text{Minimize} & \quad \alpha \\
\text{Minimize} & \quad \rho 
\end{align*}
\]
Two-step Controller

- We break the problem into two steps:
  - Turn to face the goal
Two-step Controller

- We break the problem into two steps:
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  - Move towards goal
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![Diagram of polar coordinates](image)

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- We break the problem into two steps:
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The two steps can now be specified:
- Minimize \( \alpha \)
- Minimize \( \rho \)
parameters: $k_\alpha, \epsilon_\alpha, k_\rho, \epsilon_\rho$
The Controller: Two States

parameters: $k_\alpha, \epsilon_\alpha, k_\rho, \epsilon_\rho$

$$
\begin{bmatrix}
  v(t) \\
  \omega(t)
\end{bmatrix}
= 
\begin{bmatrix}
  0 \\
  k_\alpha \text{ sign}(\alpha)
\end{bmatrix}
$$

Switch to state 2 if $|\alpha| < \epsilon_\alpha$

Note: angle $\alpha$ must be in $[-\pi, \pi]$
The Controller: Two States

parameters: \( k_\alpha, \epsilon_\alpha, k_\rho, \epsilon_\rho \)

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There are problems with this controller:

- If the first step fails, the second will also fail.
- It is difficult to choose appropriate values for the parameters: $k$, $\alpha$, $\epsilon$, $\rho$, $\epsilon$.
- Smaller thresholds require high-precision localization and actuation (if too small, goal is never reached).
- Larger thresholds reduce accuracy.
- Splitting the motion into two distinct phases is inefficient; we can save time by moving forwards while turning.
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Smooth Controller 1

- We try to minimize the quantities $g_{Rx}, g_{Ry}$, but now we minimize both simultaneously.
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- Consider the following control law:

$$v(t) = k_v g_{Rx} \omega(t)$$

$$\omega(t) = k_\omega g_{Ry}$$

where the $k$ parameters are positive.

The robot drives forward until $g_{Rx} = 0$.

If $g_{Ry}$ is positive, the robot will turn CCW to face the goal; if negative it will turn CW.
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We are given the goal pose $g_I = [g_Ix, g_Iy, g_I\theta]^T$, expressed in the inertial reference frame. 

We need the goal pose in the robot reference frame $g_R = R_{cw}(\theta)(g_I - \xi_I)$ (We should use the $3 \times 3$ rotation matrix here).
Assume we now wish to drive the robot to a desired pose.

- **Pose** means $(x, y)$ position and orientation $\theta$.

We are given the goal pose $g_I = [g_{Ix} \ g_{Iy} \ g_{I\theta}]^T$, expressed in the inertial reference frame.
Smooth Controller 2

- Assume we now wish to drive the robot to a desired **pose**
  - *Pose* means \((x, y)\) position and orientation \(\theta\)
- We are given the goal pose \(g_I = [g_{lx} \ g_{ly} \ g_{l\theta}]^T\), expressed in the inertial reference frame
- We need the goal pose in the robot reference frame
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\( g_{Rx} \alpha \rho \) \n\( g_{Ry} \) \n\( g_{R\theta} \)

The final orientation is given by \( g_{R\theta} \)
Again it will be useful to express the goal pose using polar coordinates

The final orientation is given by $g_{R\theta}$

We require a controller that minimizes $\alpha$, $\rho$, and $g_{R\theta}$ simultaneously
Consider the following control law

\[ v(t) = k \rho \omega(t) = k \alpha - k \theta g \]

where the following conditions hold:

- \( \alpha \in [-\pi, \pi] \)
- All of the \( k \) parameters are positive
- \( k \theta < k \alpha \) so \( \theta \) does not have much influence until \( \alpha \) becomes small; at this point the robot will be driven to turn away from the goal; this increases \( \alpha \) so the robot will turn towards the goal again, only now \( g R \theta \) will be reduced.

The robot drives forward until \( \rho = 0 \). If \( \alpha \) is positive, the robot will turn CCW to minimize it.
Consider the following control law

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v(t) = k_\rho \omega(t) = k_\alpha \alpha - k_\theta \theta R \theta
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If $\alpha \in (-\frac{\pi}{2}, \frac{\pi}{2}]$ then the robot will approach the goal directly (although its trajectory will be curved)
Smooth Controller 2: Refinement

- If $\alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$ then the robot will approach the goal directly (although its trajectory will be curved).
- If $\alpha \in (-\pi, -\frac{\pi}{2}] \cup \left(\frac{\pi}{2}, \pi\right]$ then the robot will first have to turn around before approaching the goal; We can detect this situation and modify the control law so that the robot backs up to the goal position, without turning around.

$$v(t) = -k_\rho \rho$$
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\begin{align*}
v(t) &= -k_\rho \rho \\
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(Here the angle $(\alpha - \pi)$ must be in $[-\pi, \pi]$)