Unit 2: Locomotion
Kinematics of Wheeled Robots: Part 1

Computer Science 4766/6912

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Kinematics?

- *Kinematics* is the study of the motion of objects without being concerned with forces
  - *Dynamics* addresses forces
- Understanding a robot's kinematics allows us to...
  - Know how a robot's body constrains its motion
  - Determine the paths and trajectories a robot can achieve
- We will address the kinematics of wheeled robots
  - Far simpler than the kinematics of legged robots
- Each wheel allows motion in some direction(s) and *constrains* it in others

Notation

- Differs somewhat from book
- Scalars in both cases: e.g. $a, B, \beta, \dot{x_R}$
- Vectors in bold lower-case: e.g. $\boldsymbol{x}, \dot{\boldsymbol{\xi}}$
- Matrices in bold upper-case: e.g. $\boldsymbol{A}, \boldsymbol{R_{ccw}}(\theta), \boldsymbol{\Phi}(t)$
- Points and axes in upper-case: e.g. P, X_R, Y_R

Representation

- We will consider only motion within the plane (i.e. 2D motion)
- The *inertial* reference frame is the global coordinate system with origin O
  - Denoted by subscript I
  - Axes X_I and Y_I
- The robot's reference frame has some origin P and describes the layout of its body w.r.t. P
  - (w.r.t. = "with respect to")
  - Denoted by subscript R
  - Axes X_R and Y_R

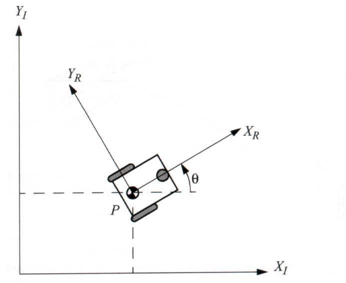

Figure 3.1
The global reference frame and the robot local reference frame.

The robot's *pose*, $\boldsymbol{\xi_I}$, gives the position of P, and the orientation of the robot's reference frame w.r.t. the inertial reference frame

$$\boldsymbol{\xi_I} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- Assume we want the robot to travel with some velocity w.r.t. the global reference frame

$$\dot{\boldsymbol{\xi}}_{\boldsymbol{I}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}$$

  where $\dot{\theta}$ causes the robot to spin about P
- We must determine how this motion vector is expressed w.r.t. the robot reference frame by rotating it back (i.e. clockwise) by θ

- Counterclockwise rotation in 2D is achieved by matrix multiplication with the rotation matrix $\boldsymbol{R_{ccw}}(\theta)$:

$$\boldsymbol{R_{ccw}}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

  (See supplementary notes on "The Rotation Matrix")
- However, we desire a clockwise rotation: $\boldsymbol{R_{cw}}(\theta) = \boldsymbol{R_{ccw}}(-\theta)$; Also, we wish to apply the rotation to $\boldsymbol{\xi_I}$ which includes a third term, θ; This term does not need to be modified by $\boldsymbol{R_{cw}}(\theta)$ so we expand $\boldsymbol{R_{cw}}(\theta)$ to leave it alone

$$\boldsymbol{R_{cw}}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Thus, the motion vector in the robot reference frame is given by

$$\dot{\boldsymbol{\xi}}_{\boldsymbol{R}} = \boldsymbol{R}_{\boldsymbol{cw}}(\theta)\dot{\boldsymbol{\xi}}_{\boldsymbol{I}} \quad (1)$$

- We will refer to the individual components of $\dot{\boldsymbol{\xi}}_{\boldsymbol{R}}$ as

$$\dot{\boldsymbol{\xi}}_{\boldsymbol{R}} = \begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix}$$

- Note that $\dot{\theta}_R = \dot{\theta}$; both describe rotation about the robot origin P

- Example: A robot which is aimed down the negative X_I axis is compelled to move in the direction $\dot{\boldsymbol{\xi}}_{\boldsymbol{I}} = [1\ 0\ 0]^T$. $\theta = \pi$, therefore

$$\dot{\boldsymbol{\xi}}_{\boldsymbol{R}} = \boldsymbol{R}_{\boldsymbol{cw}}(\theta)\dot{\boldsymbol{\xi}}_{\boldsymbol{I}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

The robot should travel along the negative X_R axis.

- A *forward kinematic model* translates the robot's actions into a motion vector in the inertial reference frame
- For most robots, an 'action' corresponds to setting the spin speeds of its wheels; e.g. spin speeds $\dot{\phi}_1$ and $\dot{\phi}_2$ for a 2-wheeled robot

Forward Kinematic Model

$$\dot{\boldsymbol{\xi}}_{\boldsymbol{I}} = f(\dot{\phi}_1, \dot{\phi}_2)$$

- An *inverse kinematic model* translates a desired motion into robot actions
- Can be difficult to obtain for some robots, e.g. manipulators
  - Many ways to achieve some poses, no solution for others

Inverse Kinematic Model

$$\begin{bmatrix} \dot{\phi}_1 & \dot{\phi}_2 \end{bmatrix}^T = g(\dot{\boldsymbol{\xi}}_{\boldsymbol{I}})$$

Wheel Kinematic Constraints

- Assumptions:
  - Plane of wheel vertical to ground
  - Single point of contact
  - No sliding or skidding
- Two Constraints:
  - Rolling constraint:
    - Motion in wheel direction = Roll speed
  - Sliding constraint:
    - Component of motion orthogonal to wheel direction = 0
  - (wheel direction $\equiv$ direction perpendicular to wheel axle)

Fixed Standard Wheel

- An unsteered wheel with a fixed position and angle w.r.t. chassis
- Located at distance l and angle α from P
- Angle of wheel w.r.t. the ray from P to wheel centre: β

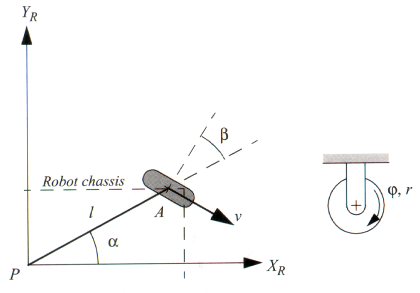

Figure 3.4
A fixed standard wheel and its parameters.

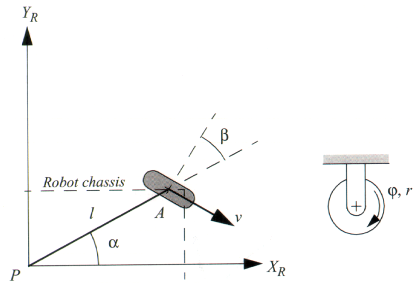

Figure 3.4
A fixed standard wheel and its parameters.

- The wheel has radius r with rotational position ϕ
- The distance that the wheel has rolled is $r\phi$
  - arc length = radius ∗ angle
- The roll speed is $r\dot{\phi}$

The Rolling Constraint

The rolling constraint for a *fixed standard wheel* (FSW) is as follows:

$$[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; (-l)\cos(\beta)]\, \dot{\xi}_R = r\dot{\phi}$$

This can be interpreted as a dot product between two vectors:

$$[\sin(\alpha+\beta) \;\; -\cos(\alpha+\beta) \;\; (-l)\cos(\beta)]$$

which describes the direction of the wheel and

$$\dot{\xi}_R$$

which is the velocity vector. Taking the dot product means finding the component of the velocity vector which is aligned with the wheel. This dot product must equal $r\dot{\phi}$ which is the roll speed.

This constraint doesn't say anything about motion orthogonal to the wheel. That's the job of the sliding constraint...

The Sliding Constraint

The sliding constraint for a FSW is as follows:

$$[\cos(\alpha+\beta) \;\; \sin(\alpha+\beta) \;\; l\sin(\beta)]\, \dot{\xi}_R = 0$$

This equation dictates that there can be no motion in the direction orthogonal to the wheel

See supplementary notes on "The Rolling and Sliding Constraints" for derivations of both

Example

- Constraints on a robot with one FSW
- COVERED ON BOARD

Constraints for Other Wheel Types

- Steered standard wheel: Differs from FSW only in that steering angle is a function of time: $\beta(t)$
  - Rolling and sliding constraints same as for FSW. Why? Constraints are instantaneous, whereas impact of changes in steering angle must be integrated over time
- Castor wheel: Given any velocity vector, we can find a roll speed and steering angle to accomodate [See book for details]
- Swedish and spherical wheels: Can accomodate any velocity vector [See book]