Unit 2: Locomotion
Kinematics of Wheeled Robots: Part 1

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

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1. Introduction
   - Kinematics?
   - Notation
   - Representation

2. Wheel Kinematic Constraints
   - Fixed Standard Wheel
   - Constraints for Other Wheel Types

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- Each wheel allows motion in some direction(s) and *constrains* it in others

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- Points and axes in upper-case: e.g. P, X_R, Y_R

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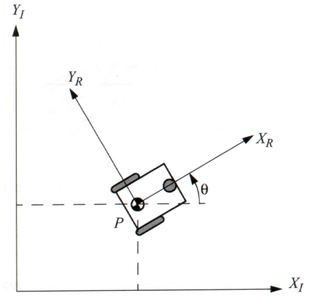

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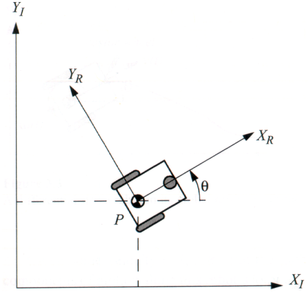
The global reference frame and the robot local reference frame.



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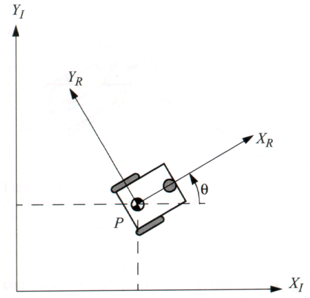

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- We must determine how this motion vector is expressed w.r.t. the robot reference frame by rotating it back (i.e. clockwise) by θ

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- Note that $\dot{\theta}_R = \dot{\theta}$; both describe rotation about the robot origin P

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The robot should travel along the negative X_R axis.

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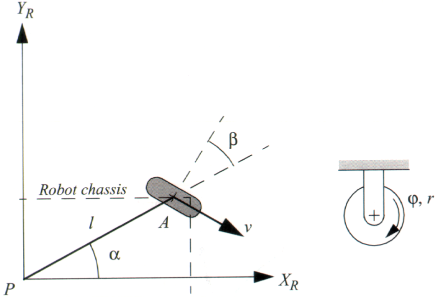
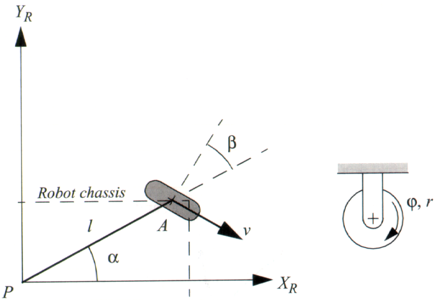


Figure 3.4
A fixed standard wheel and its parameters.



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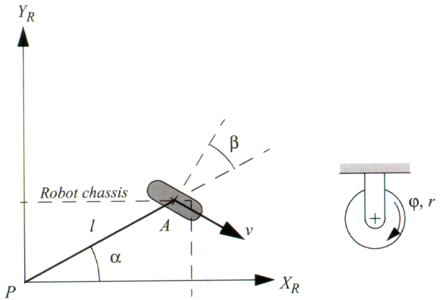

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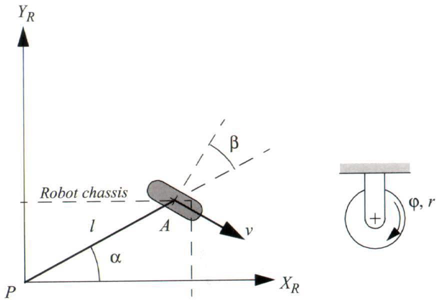

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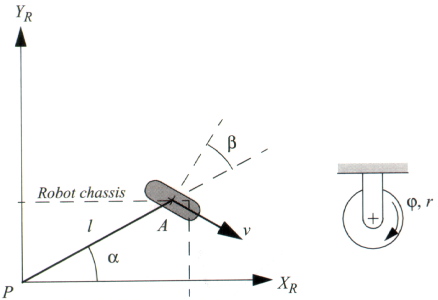

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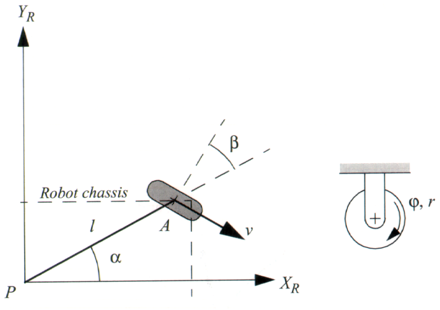

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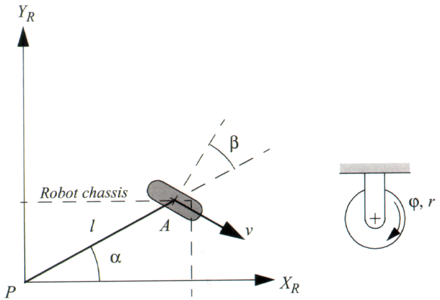

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which describes the direction of the wheel and

$$\dot{\boldsymbol{\xi}}_{\boldsymbol{R}}$$

which is the velocity vector. Taking the dot product means finding the component of the velocity vector which is aligned with the wheel. This dot product must equal $r\dot{\phi}$ which is the roll speed.

This constraint doesn't say anything about motion orthogonal to the wheel. That's the job of the sliding constraint...

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See supplementary notes on "The Rolling and Sliding Constraints" for derivations of both

Example

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- COVERED ON BOARD

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