Localization: Part 6
The Kalman Filter

Computer Science 4766/6912

Department of Computer Science
Memorial University of Newfoundland

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  - Non-linearity handled by *linearization*: **The Extended Kalman Filter**

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- If the above assumptions hold, the belief $bel(x_t)$ will always be Gaussian

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The robot begins with $\mu_0 = 0, \sigma_0 = 0$...

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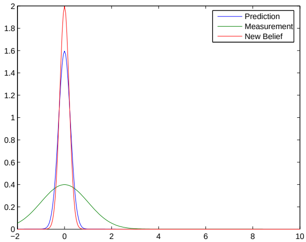

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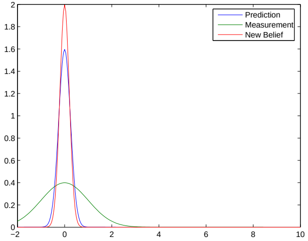

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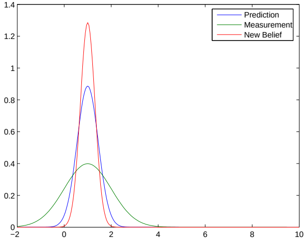

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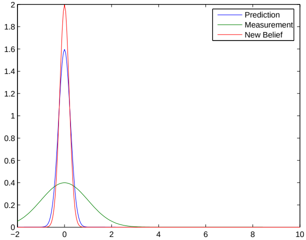

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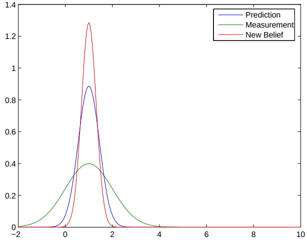

$x_t = 2 : u_t = 1,$ $z = 4(\text{error})$

$x_t = 0 : u_t = -2, z = 0$

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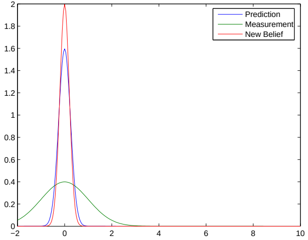

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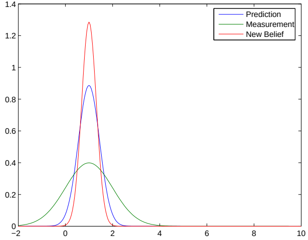

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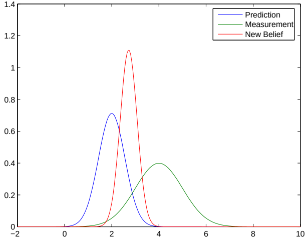

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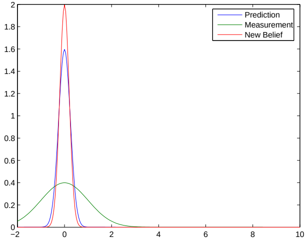

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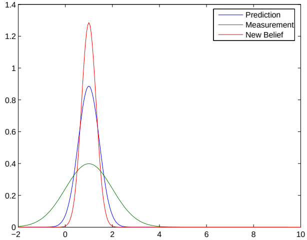

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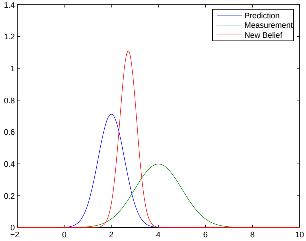

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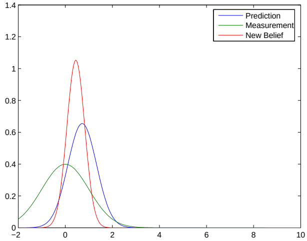

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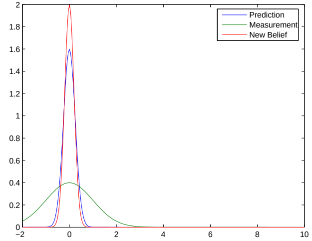

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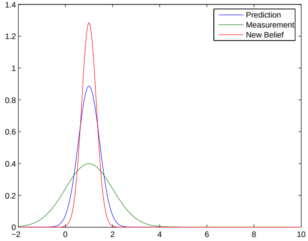

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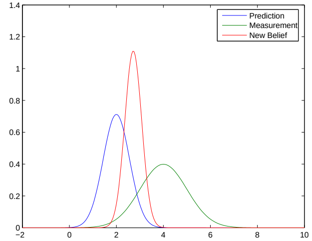

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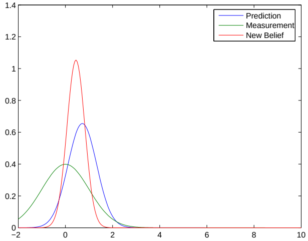

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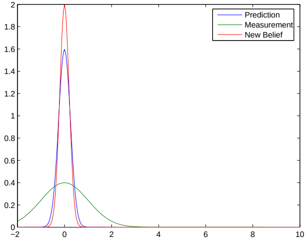

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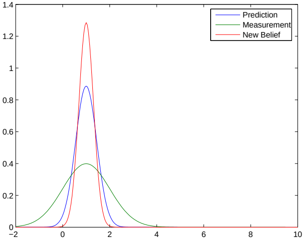

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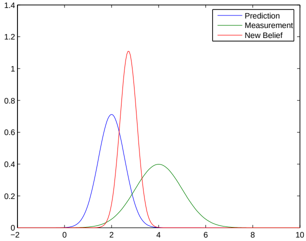

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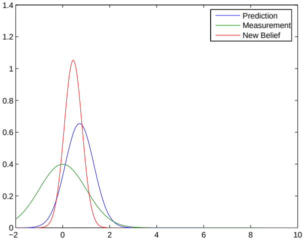

Notice when $x_t = 2$ we get an erroneous sensor value of $z = 4$. The new belief is closer to 2 than 4 because $\bar{\sigma} < q$

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Assume that this robot is also equipped with a position sensor (subject to noise of course)...

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DEMO IN MATLAB

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