Here we follow the arguments and model presented in Chapter 3 of "Swarm Robotics: A Formal Approach" by Heiko Hamann.

An essential question is to understand the relationship between the following two perspectives:

- **Microscopic:** The behaviour of individual robots
- **Macroscopic:** The behaviour of the swarm

**MICRO PERSPECTIVE**

- The micro perspective concerns the direct encoding of behaviour into the robot's controller.
- This behaviour could be encoded as a finite-state machine (FSM), neural network, decision tree, or any other similar framework.
- On the right is a FSM for a collision avoidance behaviour:
  - $s_l$ and $s_r$ are the left and right sensors and the theta values are thresholds which indicate that an object is too big (i.e., too close to the robot).

**EXAMPLE: COLLECTIVE DECISION-MAKING**

- Say there are two states: A and B
- The goal is to be in the majority state, whether it is A or B
- We define some useful quantities:
  - $N$ : Number of robots
  - $a$ : Number of robots in state A
  - $b$ : Number of robots in state B
  - (Note that $N = a + b$)
  - $\alpha = a / N$: Proportion of robots in state A
• Assume the states of neighbouring robots within a circle of radius $r$ can be determined.
• For the example on the right, this is the neighbourhood of $R_0$:
  \[ \mathcal{N} = \{R_1, R_2, R_3, R_4\} \]
• The proportion of robots in state A is
  \[ \hat{\alpha} = \frac{\hat{\alpha}}{|\mathcal{N}| + 1} \]
• Note that this includes whether the robot itself is in state A
• This is a local estimate of the global $\alpha$ which the robot can not measure directly
• If the system is “well-mixed” then perhaps $\alpha \approx \hat{\alpha}$
• But in general, $\alpha \neq \hat{\alpha}$

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**THE MICRO PERSPECTIVE**

• The robot wants to be in the same state of the majority: state A or B
• It transitions to the other state based on whether A or B is in the local majority, as indicated by this FSM:
  \[ A \quad \stackrel{\hat{\alpha} < 0.5}{\longrightarrow} \quad B \quad \stackrel{\hat{\alpha} > 0.5}{\longrightarrow} \quad \hat{\alpha} \leq 0.5 \]

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**THE MACRO PERSPECTIVE**

• First assume for simplicity that every robot has 2 neighbours: $|\mathcal{N}| = 2$
• Consider the probability of switching from state B to A, given by
  \[ P_{B \rightarrow A}(\alpha) = (1 - \alpha)\alpha^2 \]
• The first term is the probability of a robot being in state B (only robots in state B can switch to state A)
• The $\alpha^2$ term indicate the probability that both neighbours are in state A
• Similarly, we have the probability of switching from A to B:
  \[ P_{A \rightarrow B}(\alpha) = \alpha(1 - \alpha)^2 \]

• Blue curve: Probability of switching from B to A:
  \[ P_{B \rightarrow A}(\alpha) = (1 - \alpha)\alpha^2 \]
  - Decreases after maximum because with a high proportion of A robots, there are fewer available to switch
  - Decreases before maximum because with a low proportion of A robots, local majorities are less likely
Define a time step $\Delta t$ such that about one robot switches state per step.

The change in proportion of A's evolves according to this equation:

$$\frac{\Delta a(a)}{\Delta t} = \frac{1}{N}((1 - a)a^2) - \frac{1}{N}(a(1 - a)^2)$$

Left half:
- The proportion of A's will decrease, eventually reaching zero.

Right half:
- The proportion of A's will increase, eventually reaching $N$.

This simple system will always tend to favour the initial majority, but not all systems function in this way.