Here we follow the arguments and model presented in Chapter 3 of “Swarm Robotics: A Formal Approach” by Heiko Hamann.

An essential question is to understand the relationship between the following two perspectives:

- Microscopic: The behaviour of individual robots
- Macroscopic: The behaviour of the swarm
The micro perspective concerns the direct encoding of behaviour into the robot’s controller.

This behaviour could be encoded as a finite-state machine (FSM), neural network, decision tree, or any other similar framework.

On the right is a FSM for a collision avoidance behaviour:

- $s_l$ and $s_r$ are the left and right sensors and the theta values are thresholds which indicate that an object is too big (i.e. too close to the robot)
EXAMPLE: COLLECTIVE DECISION-MAKING

• Say there are two states: A and B
• The goal is to be in the majority state, whether it is A or B
• We define some useful quantities:
  • \( N \): Number of robots
  • \( a \): Number of robots in state A
  • \( b \): Number of robots in state B
  • (Note that \( N = a + b \))
  • \( \alpha = a / N \): Proportion of robots in state A
• Assume the states of neighbouring robots within a circle of radius $r$ can be determined.
• For the example on the right, this is the neighbourhood of $R_0$:

$$\mathcal{N} = \{R_1, R_2, R_3, R_4\}$$

• The proportion of robots in state A is

$$\hat{\alpha} = \frac{\hat{\alpha}}{|\mathcal{N}|+1}$$

• Note that this includes whether the robot itself is in state A
• This is a local estimate of the global $\alpha$ which the robot can not measure directly

• If the system is “well-mixed” then perhaps $\alpha \approx \hat{\alpha}$

• But in general, $\alpha \neq \hat{\alpha}$
• The robot wants to be in the same state of the majority: state A or B
• It transitions to the other state based on whether A or B is in the local majority, as indicated by this FSM:
THE MACRO PERSPECTIVE

- First assume for simplicity that every robot has 2 neighbours: $|\mathcal{N}| = 2$
- Consider the probability of switching from state B to A, given by

$$P_{B\rightarrow A}(\alpha) = (1 - \alpha)\alpha^2$$

- The first term is the probability of a robot being in state B (only robots in state B can switch to state A)
- The $\alpha^2$ term indicates the probability that both neighbours are in state A
- Similarly, we have the probability of switching from A to B:

$$P_{A\rightarrow B}(\alpha) = \alpha(1 - \alpha)^2$$
$P_{B \rightarrow A}(\alpha) = (1 - \alpha)\alpha^2$

$P_{A \rightarrow B}(\alpha) = \alpha(1 - \alpha)^2$

- Blue curve: Probability of switching from B to A:
  - Decreases after maximum because with a high proportion of A robots, there are fewer available to switch
  - Decreases before maximum because with a low proportion of A robots, local majorities are less likely
• Define a time step $\Delta t$ such that about one robot switches state per step.

• The change in proportion of A’s evolves according to this equation:

$$\frac{\Delta \alpha(\alpha)}{\Delta t} = \frac{1}{N}((1 - \alpha)\alpha^2) - \frac{1}{N}(\alpha(1 - \alpha)^2)$$

• Left half:
  • The proportion of A’s will decrease, eventually reaching zero.

• Right half:
  • The proportion of A’s will increase, eventually reaching N.

• This simple system will always tend to favour the initial majority, but not all systems function in this way.