Unit 2: Modeling in the Frequency Domain
Part 5: Modeling Translational Mechanical Systems

Engineering 5821:
Control Systems I

Faculty of Engineering & Applied Science
Memorial University of Newfoundland

January 20, 2010
Translational Mechanical Systems

- Example: Via DE
- Example: Problem Stated in the Freq. Domain
- Linearly Independent Motions
We will model mechanical systems using three components:
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- **Spring**: A spring applies a force against compression/expansion
- **Mass**: A moving mass has inertia and resists changes in velocity
- **Viscous Damper**: Resists motion (pure energy loss)
**TABLE 2.4** Force-velocity, force-displacement, and impedance translational relationships for springs, viscous dampers, and mass

<table>
<thead>
<tr>
<th>Component</th>
<th>Force-velocity</th>
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<th>Impedance ( Z_M(s) = F(s)/X(s) )</th>
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Note: The following set of symbols and units is used throughout this book: \( f(t) = \text{N (newtons)} \), \( x(t) = \text{m (meters)} \), \( v(t) = \text{m/s (meters/second)} \), \( K = \text{N/m (newtons/meter)} \), \( f_v = \text{N-s/m (newton-seconds/meter)} \), \( M = \text{kg (kilograms = newton-seconds}^2/\text{meter}) \).
Analogies with electrical quantities: Force is analogous to voltage, velocity to current, and displacement to charge.

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Analogies with electrical quantities: Force is analogous to voltage, velocity to current, and displacement to charge. Analogies with electrical components: Spring $\equiv$ Capacitor, Mass $\equiv$ Inductor, Viscous Damper $\equiv$ Resistor

**TABLE 2.4** Force-velocity, force-displacement, and impedance translational relationships for springs, viscous dampers, and mass

| Component     | Force-velocity                                      | Force-displacement                                     | Impedance  
|---------------|-----------------------------------------------------|--------------------------------------------------------|-------------
| Spring        | $f(t) = K \int_0^t v(\tau)d\tau$                   | $f(t) = Kx(t)$                                         | $K$         |
| Viscous damper| $f(t) = f_v v(t)$                                   | $f(t) = f_v \frac{dx(t)}{dt}$                         | $f_v s$     |
| Mass          | $f(t) = M \frac{dv(t)}{dt}$                         | $f(t) = M \frac{d^2x(t)}{dt^2}$                       | $M s^2$     |

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Translational Mechanical Systems

Example: Via DE
Example: Problem Stated in the Freq. Domain
Linearly Independent Motions

\[ M \frac{d^2x(t)}{dt^2} + f_v \frac{dx(t)}{dt} + Kx(t) = f(t) \]
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Just as for electrical networks, we can state the original problem in the frequency domain, as opposed to this two-step process. If we treat the displacement \( X(s) \) as the input and \( F(s) \) as the output, we can define the impedances of these components (see table).
Example: Problem Stated in the Freq. Domain

Find the transfer function $X(s)/F(s)$ for the following system,

With the understanding that each component's transfer function is its impedance $Z_M(s)$, the force from each component is $F(s) = Z_M(s)X(s)$.

We can draw the free-body diagram with forces in the freq. domain:

Summing these forces we arrive at the same result as before:

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- Assume all other independently moving parts are held still
- Draw the free-body diagram for the part, consisting of the forces due to its own motion
- Sum the forces to generate the DE or Laplace equation
e.g. Find the transfer function $X_2(s)/F(s)$ for this system,

There are two independently moving parts. Consider the forces on $M_1$ for the following two situations:

(a) Assume $M_2$ is held still and $M_1$ moves to the right

(b) Assume $M_1$ is held still and $M_2$ moves to the right

The sum of all these forces is shown in (c):
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The sum of opposing forces should equal the applied force (c):

\[
K_1X_1(s) + f_{v_1}sX_1(s) + f_{v_3}sX_1(s) + M_1s^2X_1(s) = K_2X_1(s) + f_{v_3}sX_2(s) + M_1s^2X_1(s)
\]
The sum of opposing forces should equal the applied force (c):
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\[
\left[ K_1 + K_2 + (f_{v_1} + f_{v_3})s + M_1 s^2 \right] X_1(s) - [K_2 + f_{v_3}s] X_2(s) = F(s)
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We now consider the forces on $M_2$:
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\begin{align*}
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Although complicated, we have purely a pair of linear equations to solve.
We now consider the forces on $M_2$:

\[
\begin{align*}
K_2X_2(s) &\quad (a) \\
fv_2sX_2(s) &\quad M_2 \\
fv_3sX_2(s) &\quad K_3X_2(s) \\
M_2s^2X_2(s) &
\end{align*}
\]

\[
\begin{align*}
K_2X_1(s) &\quad (b) \\
fv_3sX_1(s) &\quad M_2
\end{align*}
\]

\[
\begin{align*}
(K_2 + K_3)X_2(s) &\quad (c) \\
(fv_2 + fv_3)sX_2(s) &\quad M_2 \\
M_2s^2X_2(s) &\quad K_2X_1(s) \\
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\[
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\left[ K_2 + K_3 + (fv_2 + fv_3)s + M_2s^2 \right] X_2(s) - \left[ K_2 + fv_3s \right] X_1(s) = 0
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\end{align*}$$

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$$\frac{X_2(s)}{F(s)} = \frac{f_{v_3}s + K_2}{\Delta}$$
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where,

\[
\Delta = \begin{vmatrix}
K_1 + K_2 + (f_{v_1} + f_{v_3})s + M_1 s^2 & -(f_{v_3} s + K_2) \\
-(f_{v_3} s + K_2) & [K_2 + K_3 + (f_{v_2} + f_{v_3})s + M_2 s^2]
\end{vmatrix}
\]
Here are the two equations for the previous example again,

\[
\begin{align*}
K_1 + K_2 + (f v_1 + f v_3)s + M_1 s^2 &= F(s) \\
K_2 + (f v_3)s &= 0
\end{align*}
\]

Notice that they both adhere to the following pattern:

(∑ imped's connected to \(X_i\)) \(X_i(s)\) − (∑ imped's between \(X_i\) and \(X_j\)) \(X_j(s)\) = (∑ applied forces at \(X_i\))
Here are the two equations for the previous example again,

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\left[ K_1 + K_2 + (f_{v_1} + f_{v_3})s + M_1 s^2 \right] X_1(s) - \left[ K_2 + f_{v_3} s \right] X_2(s) = F(s)
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Notice that they both adhere to the following pattern:

\[
\left(\sum \text{imped's connected to } X_i\right) X_i(s) \\
- \left(\sum \text{imped's between } X_i \text{ and } X_j\right) X_j(s) \\
= \left(\sum \text{applied forces at } X_i\right)
\]

If there were more moving parts then there would be multiple \( X_j \) entries.
e.g. Write the equations of motion for the following system by inspection:

\[
\text{The pattern is as follows,}
\]

\[
(\sum \text{imped's connected to } X_1) X_1 (s) - (\sum \text{imped's between } X_1 \text{ and } X_2) X_2 (s) - (\sum \text{imped's between } X_1 \text{ and } X_3) X_3 (s) = (\sum \text{applied forces at } X_1)
\]
e.g. Write the equations of motion for the following system by inspection:
e.g. Write the equations of motion for the following system by inspection:

The pattern is as follows,
e.g. Write the equations of motion for the following system by inspection:

The pattern is as follows,

\[
\left( \sum \text{imped's connected to } X_1 \right) X_1(s)
\]
e.g. Write the equations of motion for the following system by inspection:

The pattern is as follows,

$$\left( \sum \text{imped's connected to } X_1 \right) X_1(s)$$

$$- \left( \sum \text{imped's between } X_1 \text{ and } X_2 \right) X_2(s)$$
e.g. Write the equations of motion for the following system by inspection:

The pattern is as follows,

\[
\left( \sum \text{imped’s connected to } X_1 \right) X_1(s)
\]

\[
- \left( \sum \text{imped’s between } X_1 \text{ and } X_2 \right) X_2(s)
\]

\[
- \left( \sum \text{imped’s between } X_1 \text{ and } X_3 \right) X_3(s)
\]
e.g. Write the equations of motion for the following system by inspection:

The pattern is as follows,

\[ \left( \sum \text{imped's connected to } X_1 \right) X_1(s) \]

\[ - \left( \sum \text{imped's between } X_1 \text{ and } X_2 \right) X_2(s) \]

\[ - \left( \sum \text{imped's between } X_1 \text{ and } X_3 \right) X_3(s) \]

\[ = \left( \sum \text{applied forces at } X_1 \right) \]
Translational Mechanical Systems

Example: Via DE
Example: Problem Stated in the Freq. Domain
Linearly Independent Motions

\[
\begin{align*}
K_1 + K_2 + \text{sf}_v v_1 + \text{sf}_v v_3 + M_1 s^2 X_1(s) &= 0 \\
-K_2 X_2(s) - \text{sf}_v v_3 X_3(s) &= F(s) \\
\text{sf}_v v_3 X_1(s) - \text{sf}_v v_3 X_2(s) + \text{sf}_v v_4 X_3(s) &= 0
\end{align*}
\]
Translational Mechanical Systems

Example: Via DE
Example: Problem Stated in the Freq. Domain
Linearly Independent Motions

\[
[ K_1 + K_2 + s f_{v_1} + s f_{v_3} + M_1 s^2 ] \ X_1(s)
\]
\[
\begin{bmatrix}
K_1 + K_2 + sf_{v_1} + sf_{v_3} + M_1 s^2
\end{bmatrix} X_1(s) - [K_2] X_2(s)
\]
Translational Mechanical Systems

Example: Via DE

Example: Problem Stated in the Freq. Domain

Linearly Independent Motions

\[
\begin{bmatrix}
K_1 + K_2 + s f_{v_1} + s f_{v_3} + M_1 s^2
\end{bmatrix} X_1(s) - \begin{bmatrix} K_2 \end{bmatrix} X_2(s) - \begin{bmatrix} s f_{v_3} \end{bmatrix} X_3(s) = 0
\]
Translational Mechanical Systems

Example: Via DE
Example: Problem Stated in the Freq. Domain
Linearly Independent Motions

\[
\begin{bmatrix}
K_1 + K_2 + sf_{v_1} + sf_{v_3} + M_1 s^2 \\
K_2 \\
f_{v_3} \\
M_2
\end{bmatrix} X_1(s) \quad - \quad \begin{bmatrix}
K_2 \\
f_{v_4} \\
f_{v_2}
\end{bmatrix} X_2(s) \quad - \quad [sf_{v_3}] X_3(s) = 0
\]

\[ - [K_2] X_1(s) \]
\[
\begin{align*}
\left[K_1 + K_2 + s f_{v_1} + s f_{v_3} + M_1 s^2\right] X_1(s) - [K_2] X_2(s) - [s f_{v_3}] X_3(s) &= 0 \\
- [K_2] X_1(s) + [K_2 + s f_{v_2} + s f_{v_4} + M_2 s^2] X_2(s) &
\end{align*}
\]
Translational Mechanical Systems

Example: Via DE
Example: Problem Stated in the Freq. Domain
Linearly Independent Motions

\[
\begin{align*}
[K_1 + K_2 + sf_{v_1} + sf_{v_3} + M_1 s^2] X_1(s) - [K_2] X_2(s) - [sf_{v_3}] X_3(s) &= 0 \\
-[K_2] X_1(s) + [K_2 + sf_{v_2} + sf_{v_4} + M_2 s^2] X_2(s) - [sf_{v_4}] X_3(s) &= F(s)
\end{align*}
\]

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\[
\begin{bmatrix}
K_1 + K_2 + sf_{v_1} + sf_{v_3} + M_1 s^2
\end{bmatrix} X_1(s) - \begin{bmatrix} K_2 \end{bmatrix} X_2(s) - \begin{bmatrix} sf_{v_3} \end{bmatrix} X_3(s) = 0
\]

\[
- \begin{bmatrix} K_2 \end{bmatrix} X_1(s) + \begin{bmatrix} K_2 + sf_{v_2} + sf_{v_4} + M_2 s^2 \end{bmatrix} X_2(s) - \begin{bmatrix} sf_{v_4} \end{bmatrix} X_3(s) = F(s)
\]

\[
- \begin{bmatrix} sf_{v_3} \end{bmatrix} X_1(s)
\]
\[
[K_1 + K_2 + sf_{v_1} + sf_{v_3} + M_1 s^2] X_1(s) - [K_2] X_2(s) - [sf_{v_3}] X_3(s) = 0
\]

\[
-K_2 X_1(s) + [K_2 + sf_{v_2} + sf_{v_4} + M_2 s^2] X_2(s) - [sf_{v_4}] X_3(s) = F(s)
\]

\[
-sf_{v_3} X_1(s) - sf_{v_4} X_2(s)
\]
Translational Mechanical Systems

Example: Via DE
Example: Problem Stated in the Freq. Domain
Linearly Independent Motions

\[
\begin{align*}
\begin{bmatrix}
K_1 + K_2 + s f_{v_1} + s f_{v_3} + M_1 s^2
\end{bmatrix} X_1(s) - \begin{bmatrix} K_2 \end{bmatrix} X_2(s) - \begin{bmatrix} s f_{v_3} \end{bmatrix} X_3(s) &= 0 \\
- \begin{bmatrix} K_2 \end{bmatrix} X_1(s) + \begin{bmatrix} K_2 + s f_{v_2} + s f_{v_4} + M_2 s^2 \end{bmatrix} X_2(s) - \begin{bmatrix} s f_{v_4} \end{bmatrix} X_3(s) &= F(s) \\
- \begin{bmatrix} s f_{v_3} \end{bmatrix} X_1(s) - \begin{bmatrix} s f_{v_4} \end{bmatrix} X_2(s) + \begin{bmatrix} s f_{v_3} + s f_{v_4} + M_3 s^2 \end{bmatrix} X_3(s) &= 0
\end{align*}
\]

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