Unit 3: Time Response
Part 1: Poles and Zeros and First-Order Systems

Engineering 5821:
Control Systems I

Faculty of Engineering & Applied Science
Memorial University of Newfoundland

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1 Poles and Zeros

1 First-Order Systems
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The poles and zeros of the input and the transfer function can be quickly inspected to determine the form of the system response. Of course, we can obtain this form from the ILT, but looking at the poles and zeros allow us to see it more quickly.
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Sometimes we also classify as zeros or poles roots of the denominator (poles) or numerator (zeros) which are common and can therefore be cancelled. These so-called zeros or poles lack the ability to make the function go to zero or infinity, yet are sometimes referred to as zeros or poles nevertheless.
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The following figure illustrates the relationship between the system response and the poles and zeros...
\[ R(s) = \frac{1}{s} \]

\[ G(s) = \frac{s + 2}{s + 5} \]

\[ C(s) = \]

(a)

(b)

Input pole
\[ \frac{1}{s} \]

System zero
\[ \frac{s + 2}{s + 5} \]

System pole
\[ \frac{1}{s + 5} \]

\[ j\omega \]

\[ s - \text{plane} \]

\[ \sigma \]

Output transform
\[ C(s) = \frac{2/5}{s} + \frac{3/5}{s + 5} \]

Output time response
\[ c(t) = \frac{2}{5} + \frac{3}{5}e^{-5t} \]

(c)

Forced response

Natural response
Notice the following:

- The input pole generates the form of the forced response.
- The system pole generates the form of the natural response.
- This pole is on the negative real axis. Hence, it generates a decaying exponential.
- The zeros and poles together contribute to the calculation of $A$ and $B$.

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![Diagram of the s-plane with a pole at -\(\alpha\) and the corresponding response formula \(K_0e^{-\alpha t}\).]
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**Diagram:**

- Pole at $-\alpha$ generates response $Ke^{-\alpha t}$ in the $s$-plane.
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**e.g.** What is the form of the system response for the following system?

\[
R(s) = \frac{1}{s} \quad \frac{(s + 3)}{(s + 2)(s + 4)(s + 5)} \quad C(s)
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The input pole (at \( s = 0 \)) generates the forced response \( c_f(t) = 1 \) while the system pole yields the natural response \( c_n(t) = -e^{-at} \).
We can rewrite this function as follows,

\[ c(t) = 1 - e^{-at} = 1 - e^{-t/\tau} \]

where \( \tau = \frac{1}{a} \) is the time constant.

At the time constant the function reaches 63% of its final value.

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\[ C(s) = Ks \frac{a}{s + a} = Ka \frac{s}{s + a} \]

Determine \( a \) and \( K \).

The asymptote yields an estimate of \( K/a \approx 0.72 \).

63% of this is 0.45 which is reached around \( t = 0.15 \).

Hence \( a = 1/0.15 = 6.67 \) and \( K = 4.8 \).
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![Diagram showing step response with asymptote and estimated values for K and a.](image)
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