On the Dynamics of Competition in a simple Artificial Chemistry

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We examine a simple system of competing and cooperating entities in terms of the speed of settling their competition. It turns out that the larger the degree of cooperativity among entities the quicker the competition is decided. This result, derived in a simple artificial chemistry system, demonstrates that cooperativity is a decisive element of a world of entities competing for resources. It also hints at the fact that growth of complexity (in terms of increasing cooperativity) is a native tendency of such a world.

Key words: artificial chemistry, competition, cooperation, dynamics, accelerated evolution

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1 Introduction

Artificial Chemistries (ACs) are model systems for the study of phenomena in, among other fields, Physics, Chemistry, Biology, Economy and Sociology [1]. They make use of the fact that, in computer simulations, one is free to define arbitrary interaction rules between objects of arbitrary behavior. Thus, it is possible to define, for instance, algorithmic "atoms" and their interaction rules for the purpose of studying one specific question in simulation, such as their bonding behavior when equipped with an interaction force alternative to what is presently considered in physics.

The entities of ACs might be quite complicated, allowing them to play the role of, e.g., agents in a multi-agent system for the study of social questions. Whereas in multi-agent systems the emphasis lies in studying individual behavior of agents, ACs are used to draw conclusions about collective behavior. In the case of a social simulation, for instance, entities might be equipped with adaptive capabilities like learning, allowing them to be influenced by collective phenomena that appear on a mesoscopic or macroscopic level. This makes them an ideal tool to examine complex systems. In the context of this paper, we define complex systems by the existence of feedback loops from the emergent levels of system behavior back to the "atomic" level of the system's defining entities. The existence of this feedback between levels (there might be more than two levels) often renders analytical approaches powerless since the degree of non-linearity of such systems is enormous.

There are notable exceptions from this inability to treat systems analytically. Synergetics [2, 3] has devoted most of its efforts to studying complex systems which show regularities in their behavior that can be derived from certain assumptions about the interaction between levels. The slaving principle is a typical example of a concept developed when studying the feedback between physical entities and their collective state, e.g., excited atoms and the radiation in a laser system [4]. Being able to treat systems analytically comes at a price, however. It requires that the entities are relatively simple behaved, such as atoms, and that the feedback between levels is relatively simple, like inhibitory or excitatory. As soon as combinatorics comes into play when defining the interaction between entities, as it is the case in, e.g., Chemistry, the feedback between levels is not easily derived and surprising

effects might happen. It is at this stage of things becoming complicated that ACs come into their own.

As a simulation tool, ACs cannot substitute for analytical treatment, however, and some connection between the two approaches must be maintained if one is to learn from AC models. In a large class of ACs, this is the case, and systems of coupled nonlinear differential equations, to name one prominent method, can be used to predict the behavior of a simulation very well. Though this capability is restricted to low-dimensional models, it nevertheless provides a valuable bridge between traditional methods and this new approach.

One last aspect deserves mentioning, although we cannot elaborate on this aspect in the model system below. It is the aspect of "constructivity" of ACs and many natural systems. Whereas Synergetics assumes the entities as given, and their interaction as fixed, ACs are able to work up the dynamics of a system from the point of a seed system with only a few entities. It can be said that, in the course of their dynamics, the systems construct themselves. For instance, ACs have no difficulty in allowing new entities encountered never before into the system and participating in the dynamics. By the new entries, new interactions might come into play that, again, were encountered never before. This way, systems of growing complexity can be simulated and conclusions can be drawn from their behavior. Many natural systems are constructive, e.g. elementary particle physics, chemistry, ecosystems, development of organisms, societies. The field of constructive systems is just in its beginnings and we shall therefore refrain from discussing it in more detail here.

The question we want to approach with the model introduced here is the following: How does the dynamics of a competitive system change if cooperative interactions are added to the competitive interactions. This question is of interest since it might help to explain the development of cooperation between entities. In the light of the aforementioned self-construction of systems, cooperation is the precondition for the transition to higher levels of complexity in a system, and thus of growing complexity of a qualitative type.

The paper is organized as follows: Sec 2 will introduce the model and its analytical counterpart, Sec 3 will show simulation results, Sec 4 connects to the behavior of the differential equation system. Sec 5 puts the behavior into context and draws conclusions.

2 The Model

Following [1] we define an artificial chemistry by introducing its objects and their interaction and by determining the dynamics of the system. Here, our entities are objects of N non-combinatorial types (types) $s_i, i = 1, ..., N$ which interact catalytically when encountering each other. The fact that objects are non-combinatorial allows us to connect the dynamics to the more traditional method of coupled differential equations. An interaction between objects can be written down in a chemical description as

$$s_i + s_j + [X] \rightarrow s_i + s_j + s_k \; . \tag{1}$$

The notion is that, once two objects encounter each other, they produce a third object under the assumption that the consumption of energy and material ([X]) both can be neglected. Interactions take place in a continuously-stirred reaction vessel of finite size M. Thus, in order to avoid an increase in the number of objects, each time a new object is produced another object will be removed from the vessel. This defines the dynamics and results in a continuous flow out of the reactor.

In order to fix the interaction between objects, a reaction matrix $R_{i,j}$ can be defined that determines which objects are produced (entries in the matrix) upon the collision of which entities (columns and rows of the matrix, see equ. 3). An equivalent formulation is a three-dimensional matrix $W_{i,j,k}$ which assumes only two values, "1" if k is produced by the interaction of i, j, "0" if this is not the case.

Matrix W is useful in a rate equation description of the system. For this purpose we must assume that a very large number of copies of each species $s_i, i = 1, ..., N$ is present in the reaction vessel. The large number of objects allows to neglect random fluctuations that come about by the order in which

reactions happen in the vessel or by the possibly low numbers of copies of a species leading to extinction events. The rate equation model for the system can be written down as follows:

$$\frac{dx_i}{dt} = \sum_{j,k=1}^{N} W_{ijk} x_j x_k - \Phi x_i \ , i = 1, \ldots, N. \quad (2)$$

where the flow term with Φ ensures that the sum of concentrations remains 1. These equations can be solved numerically, and describe the system deterministically. If care is taken (sufficiently small numerical constants for integration), both methods, explicit simulation of the AC and numerical solution of its rate-equation solution should converge to the same answers.

For the purpose of this contribution we shall consider $N = 7$ different types of objects. Figure 1 (a-l) is a visualization of the interaction between object types. Mathematically, these graphs correspond to reaction matrices. For instance, the reaction matrix corresponding to the situation of Figure 1(f) can be written as

$$\begin{pmatrix} 1 & 0 & 3 & 0 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 4 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7 \end{pmatrix}. \quad (3)$$

What we are interested in is the comparative dynamical development of these systems with a different degree of competition and cooperation. While the first model basically contains autocatalytic reactions (each type produces itself only), larger and larger numbers of interactions are cooperative in the sequence of models. So we have 7, 6, 5, 4, 3, 2 competitions with a corresponding growing number of cooperative interactions.

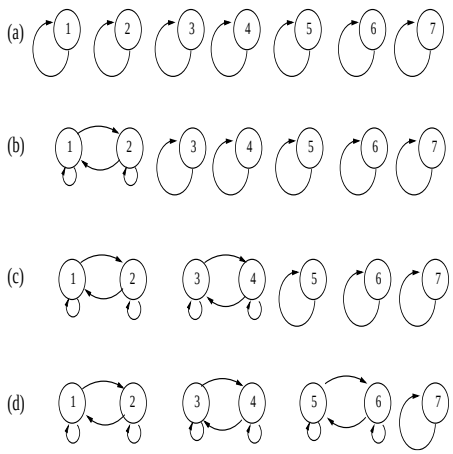

FIG. 1. Competition of 7 types of entities. (a) Each entity is autocatalytic only and competes with all others for resources; (b) Two entities are supporting each other and compete as a pair, all others are pure autocatalysts; (c) Two groups of entities support each other; (d) three groups of two entities each and one autocatalyst.

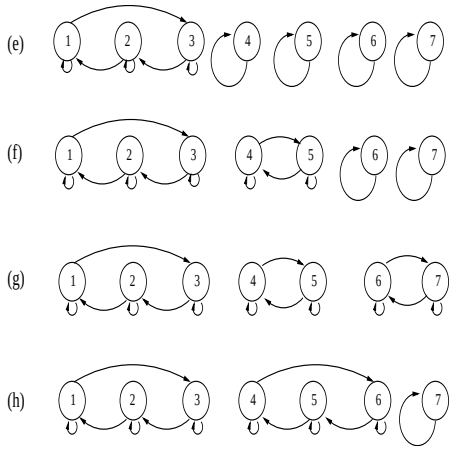

FIG. 1. (e) A larger group of three entities supports itself in cyclic fashion, 4 autocatalysts compete; (f) a group of three, one of two and two autocatalysts; (g) two groups of 2 and one group of three compete; (h) two groups of three each, and one autocatalyst.

3 Behavior of the Model

Given a set of $N = 10^4$ objects of the different types, what is the behavior of the model? Figure 2 shows a typical example of the development of concentrations over time. As can be seen, the dynamics is indeed competitive and one entity (or a collective) achieves full concentration in the reactor. The nonlinear reactions between types, together with the flux term allow for no other solution. Note that in the case of a collective winning the competition there is a continued shift in concentrations between the participating types (see Figure 3). This shift is

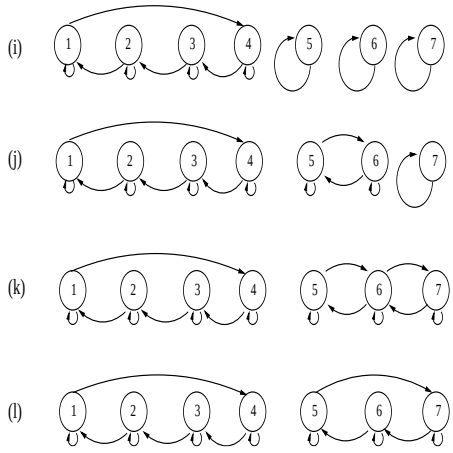

FIG. 1. (i) One group of four entities competes with three autocatalysts; (j) one group of four, one group of two and an autocatalyst compete; (k) a group of four competes against a group of three.

mostly triggered by fluctuations in concentrations, and is less emphasized in a reactor with more objects.

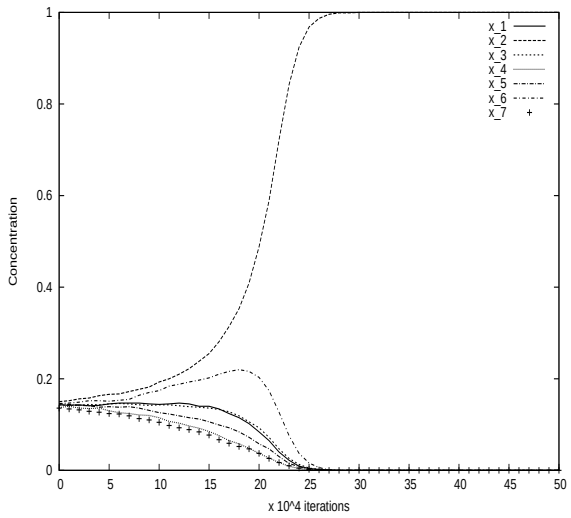

FIG. 2. Typical concentration development in a simulation with 10^4 objects in the reactor. Competition of Figure 1(a). Largest concentration at the outset wins competition.

We shall now compare different degrees of cooperation/competition in terms of the speed of settlement of the competition . For this purpose we need a termination criterion for competitions [5]. We de-

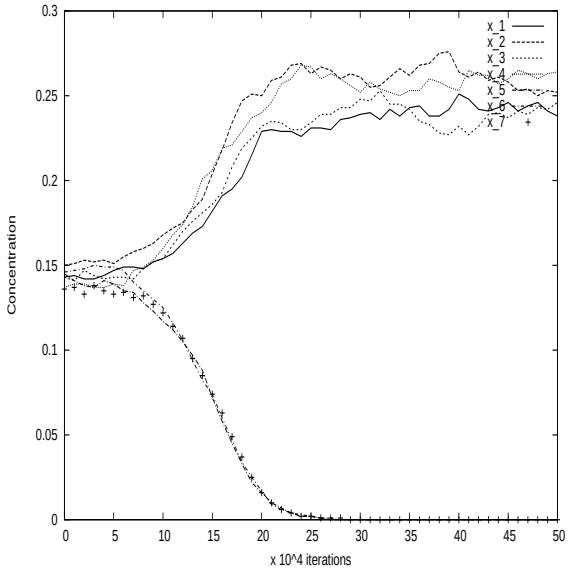

FIG. 3. Typical concentration development in a simulation with 10^4 objects in the reactor. Competition of Figure 1(l). The collective of 4 cooperating entities wins competition.

fine x_L as the concentration of all losing entities i_L, lumped together,

$$x_L = \sum_{i_L} x_{i_L} \qquad \forall i_L \tag{4}$$

Applying the following criterion:

$$x_L \leq 10^{-2} \tag{5}$$

we can measure the number of iterations necessary for termination to occur. This number can be compared for various interaction types (see Figure 1 (a-l)) and averaged over many different initial conditions.

Figure 4 shows a sample run clearly exhibiting different speed of competition until settlement for a specific initial condition. Table 1 lists the average number of iterations until settlement for the various situations of Figure 1, together with the number of competitive and cooperative interactions present in those situations.

The general tendency of the dynamics can be read off from Table 1. As soon as cooperation comes into

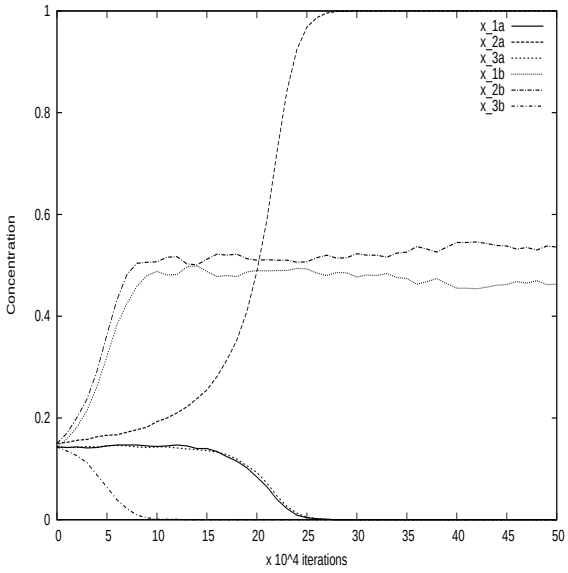

FIG. 4. Comparison of settlement speed in a typical simulation run with 10^4 objects in the reactor. Competition of dynamics of Figure 1(a) (x_{1a}, x_{2a}, x_{3a}) vs. dynamics of Figure 1(b) (x_{1b}, x_{2b}, x_{3b}). The collective of 2 cooperating entities wins competition much earlier.

Figure	Comp	Coop	Iterations	Std. Dev.
1a	7	0	39,2	3.4
1b	6	1	10.0	0.0
1c	5	2	18.1	1.4
1d	4	4	21.7	2.4
1e	5	3	12.0	0.2
1f	4	5	14.0	0.0
1g	3	7	24.9	4.7
1h	3	6	25.9	3.4
1i	4	4	12.7	0.5
1j	3	6	22.2	2.3
1k	2	8	70.3	16.7
1l	2	7	26.5	2.6

Table 1. Comparison of the computational costs for 100 runs in each configuration of Fig 1 (a-l). Average and std. deviation of number of iteration steps given. Comp: Number of competitions; Coop: Number of cooperations.

play, the dynamics settles faster. There is one exception, which is due to an artefact. The situation of Figure 1 (k) takes more time to settle, because there is an intermediate state of the system where concentrations are high in species 5 to 7. This later (in iteration 70, on average) develops into a single species assuming concentration 1. One can also notice that configurations settle even more quickly than others. There are basically two causes here for the difference: (i) the number of competitions and cooperations in a configuration, (ii) the difference between the winning and loosing parts of the species population at the outset. This latter reason causes a quicker settlement of the dynamics the larger the difference at the outset. Thus, configurations of Figure 1(b), (e) and (i) have an advantage.

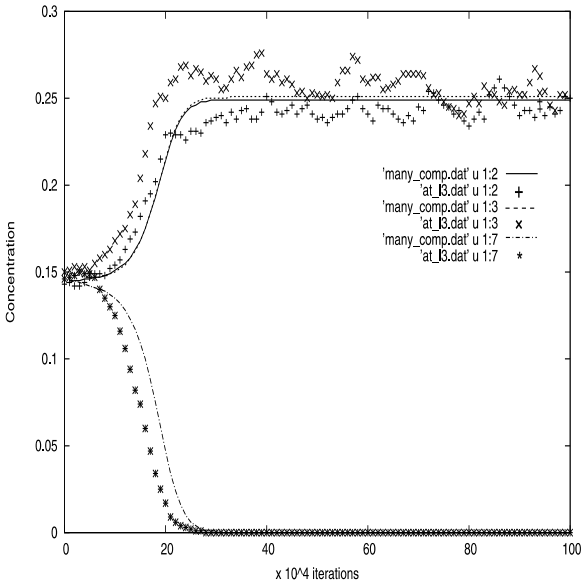

FIG. 5. Comparison of dynamics for Figure 1(l) with a reactor of $M = 10^4$ objects and rate equations.

The differential equation system of eqns. (2) can be integrated numerically. Figure 5 shows an overlay of the same situation for the dynamics of a discrete simulation and a direct numerical integration. It turns out that there is good coincidence between the two methods. This had been observed in other models of similar type in the past [6].

4 Consequences

We have seen that in our model world the degree of cooperation or mutualism influences the speed of the decision dynamics. It might not be far-fetched

to suspect that similar mechanisms are at work in the natural world. There, we observe an increasing degree of cooperativity (above the level of replicators). Due to the fact that new entities emerge by the cooperation of lower-level entities, it can be said that cooperation fosters the construction of hierarchies within a system, and thus enables the growth of complexity. In other words, the development of cooperative interaction adds a competitive edge to the participating units and accelerates competition dynamics with others. In this way not only hierarchies are constructed, but the dynamical evolution of the system accelerates.

The emergence of hierarchies can indeed be observed by the spread in time scales of development in a system. Hierarchical levels are, among other things, differentiated by the speed by which they undergo changes. As long as there are more or less the same time scales of change for entities, we cannot speak of different levels of a hierarchy. Once a separation in time scales occurs, with the lower level of the hierarchy developing slower than the higher level (this reflects the fact of acceleration) levels have separated.

It is interesting to note that in artificial systems, the formation of hierarchical levels can be enforced by determining the time scale of change of entities at the different levels. There, two alternative approaches are available: (i) either one can accelerate the dynamics of change at the higher level, or (ii) one can slow down the dynamics of change at the lower level [7]. In biological systems both approaches have been adopted by Nature. The evolution of the genetic code [8] has virtually come to a stand-still (some researchers even stated it would be frozen [9]) in order to allow higher levels of the evolutionary hierarchy to rely on it. At the level of genomes a number of repair mechanisms are employed by organisms to ensure robustness against copy errors for DNA [10]. This way Nature solidified the separation of hierarchical levels in the respective systems. Once the speed of change has been minimized at the lowest level, however, only acceleration at higher levels allows to build further hierarchical levels. It might be ventured that this is the reason we observe acceleration with the emergence of further hierarchical levels.

To put these results into perspective: H.A.Simon has pointed out [11, 12], that hierarchies are strongly correlated with modularity, and modularity or (near-) decomposability with speed of evolution. The more hierarchic a system is, (a) the better it can absorb perturbations and (b) the faster it can evolve. The former aspect has a long tradition and has been recently examined in a simple model ecosystem [13] where it was shown that robustness increases with the degree of mutualism. A biological result to the same effect has been demonstrated earlier [15]: there it has been shown that, upon transition to mutualism, evolutionary change can accelerate without harming the organisms. The latter aspect is perhaps less well-known in the community.

5 Summary and Conclusion

We have shown that a very simple system of interacting entities suffices to demonstrate acceleration effects of a dynamics equipped with cooperative interactions. Besides autocatalytic interactions, a number of different dynamics have been examined in a system with 7 different types of objects. The general tendency was an increase in the settlement speed for competition due to the fact that more entities of the "universe" were involved per competition.

It was pointed out that this result is in line with observations both from the non-living and living natural world.

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